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AUTHOR DeVenney, William S.; And Others
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ABSTRACT

This text is one of the sequence of textbooks produced for low achievers in the seventh and eighth grades by the School Mathematics Study Group (SMSG). There are eight texts in the sequence, of which this is the last. This set of volumes differs from the regular editions of SMSG junior high school texts in that very little reading is required. Concepts and processes are illustrated pictorially, and many exercises are included. This volume continues the study of equations begun in chapter 8, and develops methods for solving linear inequalities and quadratic equations. In the last chapter the slope-intercept form of a linear equation is discussed, and the method of solution of simultaneous linear equations is detailed. Equations of parallel and perpendicular pairs of lines are examined. The concepts of absolute value and distance are introduced, and the method of computing the distance between two points in a plane is described. (SD)

SECONDARY SCHOOL MATHEMATICS

SPECIAL EDITION

Chapter 17. Solving Equations and Inequalities

Chapter 18. Coordinate Geometry

MEMBERS OF WRITING TEAMS

William S. DeVenney, Ashland, Oregon
James C. McCaig, Cupertino School District, Cupertino, Calif.
Jane G. Stenzel, Cambrian Elementary School District, San Jose, Calif.

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SOLVING EQUATIONS AND INEQUALITIES

Introduction

In English it is possible to write statements that are not true as well as statements that are true. When you say, "This cat is an animal with four legs," the statement is true if the thing you are pointing to is a normal cat. On the other hand, it is not true if you say, "An airplane is an animal with four legs."

In your arithmetic work in the lower grades, you probably gave "wrong" answers to some problems once in a while. When this happened, you had written a mathematical statement that was not true.

Many tests have sentences with blanks where you are supposed to write a word or two to complete the sentence. If you write the correct word, the sentence is a true statement. If you write the wrong word, the statement is false. For instance, in the following:

> "__________ was the first President of the United States.",

"Washington" makes the statement true. "Lincoln" makes it false.

In mathematics, just as in English, there are different kinds of sentences. When you say $\overline{AB} \perp \overline{CD}$, or $\triangle ABC \cong \triangle XYZ$, you are making statements which may or may not be true, but they are different kinds of statements from

$$3 + 2 = 5 .$$

The last statement says that the number named on the left side of the equal sign is the same number as the number named on the right side. That is, "The sum, three plus two, is five."

Suppose we replace the 3 in the statement above with a variable, x :

$$x + 2 = 5 .$$

This <u>equation</u> is a sentence that claims there is a number with which we can replace the variable, x, to make a true statement. The <u>solution</u> of the equation is the number used to make it true. In $x + 2 = 5$, <u>the</u> solution is 3. No other number makes the sentence true. (Of course you know that there are many names for 3: $\frac{6}{2}$, 3×1, $3 + 0$, $4 - 1$, etc., but it is easiest to recognize the ordinary name, 3, and that is the one you would expect to see in the solution.) For some equations there is <u>no</u> solution. For instance, there is no real number to replace x in the equation $x = \sqrt{^-4}$ and make the sentence true.

An equation can be so simple that its solution is perfectly obvious. In $x = 2$, for instance, you see at once that replacing the variable with 2 makes the sentence true: $2 = 2$.

Some equations, however, are not so easy to solve. In this chapter you will learn ways to solve equations like these: $3x + 4 = 2x + 1$, and $x^2 + 4x + 4 = 1$.

You will see that in some equations more than one number is a solution. You will also find out how to solve <u>inequalities</u> like $x + 3 < 5$.

<u>Class Discussion</u>

In order to solve equations it is helpful to look first at mathematical statements without variables.

Take a true statement like

$$3 + 4 = 7.$$

Suppose you multiply the number $(3 + 4)$ on the left side of the equal sign by 2. (We will call this multiplying "the left side" by 2.)

$2 \cdot (3 + 4) = 7$. This ______ (is, is not) a true statement. You can leave the left side as it is and make a true statement again by ______ the number on the right side by ______.

$$2 \cdot (3 + 4) = \_\_\_\_ \cdot 7$$

<u>If you multiply both sides of a true statement by the same number, you still have a true statement.</u>

Suppose you add 5 to the left side of the statement $3 + 4 = 7$. You get $(3 + 4) + 5 = 7$. This ________ (is, is not) a true statement.

You can leave the left side as it is and make a true statement again by ______ ____ to the right side.

$$(3 + 4) + 5 = 7 + \_\_\_$$

<u>If you add the same number to both sides of a true statement, you still have a true statement.</u>

1. Show what you do to the second statement of each pair below to make each statement true.

(a) $5 + 9 = 14$

$3 \cdot (5 + 9) = \_\_\_\_\_\_ \; 14$

(b) $4 + 6 = 10$

$(4 + 6) + 3 = 10 \; \_\_\_\_\_\_$

(c) $26 + 8 = 34$

$9 \cdot (26 + 8) = \_\_\_\_\_\_ \; 34$

(d) $15 + 45 = 60$

$3 \cdot (15 + 45) + 8 = \_\_\_\_ \cdot 60 + \_\_\_\_$

Some things that you do in everyday life you can "undo". To undo putting on your coat, you ______________. To undo picking up a book, you ______________. To undo opening the door, you ______________.

In working with numbers, you also can "undo" what you do to a number.

To undo adding 5, you add the ________ of 5.

If you start with 3 and add 5, you get 8.

$$3 + 5 = 8$$

To get back to 3 again, you add ________ to <u>both</u> <u>sides</u> of the equation.

$$3 + 5 + {}^{-}5 = 8 + {}^{-}5$$

You have a true statement: $3 + 0 = 3$

or simply: $3 = 3$.

Suppose you use a variable in place of the 3 in the equation you started with. If you write $x + 5 = 8$ then to find out what x is you add the opposite of 5 to <u>both</u> <u>sides</u> of the equation.

$$x + 5 + {}^{-}5 = 8 + {}^{-}5$$

On the left side, adding the numbers, you have $x + 0$, and on the right side you have ________.

$$x + 0 = 3$$

so $\quad x = 3$

2. In the equation $x + {}^{-}16 = 24$, to undo adding ${}^{-}16$, you must add ________ to both sides of the equation.

$$x + {}^{-}16 + 16 = 24 + \_\_\_\_\_\_$$

so $\quad x =$ ________

3. In the equation $x + 59 = 65$, to find out what x is you add ________ to both sides of the equation.

$$x + 59 = 65$$

$$x + 59 + \_\_\_\_ = 65 + \_\_\_\_$$

$$x + 0 = \_\_\_\_$$

$$x = \_\_\_\_$$

To undo <u>multiplying</u> by 5, you multiply by the <u>reciprocal</u> of ________.

If you start with 4 and multiply by 5, you get 20. To get back to 4 again, you multiply <u>both</u> <u>sides</u> of the equation by ________.

$$5 \cdot 4 = 20$$

$$\frac{1}{5} \cdot 5 \cdot 4 = \_\_\_\_ \cdot 20$$

$$1 \cdot 4 = \_\_\_\_$$

so $$4 = \_\_\_\_$$

Suppose you have a variable in place of the 4 in the first equation.

$$5 \cdot x = 20$$

You multiply both sides of the equation by $\frac{1}{5}$.

$$\frac{1}{5} \cdot 5 \cdot x = \frac{1}{5} \cdot 20$$

so $$1 \cdot x = 4$$

and $$x = 4$$

4. In the equation $10x = 90$, to find out what x is you multiply both sides of the equation by _____. (Remember that $10x$ means $10 \cdot x$.)

$$10x = 90$$

$$\frac{1}{10} \cdot 10x = \_\_\_\_ \cdot 90$$

$$1 \cdot x = \_\_\_\_$$

so $$x = \_\_\_\_$$

5. In the equation $\frac{1}{4}x = 12$, you multiply both sides of the equation by _____.

$$\frac{1}{4}x = 12$$

$$\_\_\_\_ \cdot \frac{1}{4}x = \_\_\_\_ \cdot 12$$

$$1 \cdot x = \_\_\_\_$$

so $$x = \_\_\_\_$$

What we want to do in solving equations is to add the same number to both sides of the equation or multiply both sides by the same number in order to get just the variable on one side and the solution on the other. As we do this, we write a chain of equivalent equations, all of which have the same solution.

In the equation $x + 9 = {}^{-}3$, we want to get $x + 0 = ?$, so we add ______ to both sides, and learn that $x =$ ______ .

In the equation $\frac{1}{5} x = 3$, we would like to have $1 \cdot x = ?$, so we multiply both sides by ______ and learn that $x =$ ______.

Exercises

For each of the following, show what you do to both sides of the equation to find the solution. Then solve the equation. The first one is done for you.

Equation	On both sides, you
1. $x + 6 = 13$ $x = \underline{7}$	add ${}^{-}6$.
2. $x + {}^{-}5 = 3$ $x =$ ______	add ______ .
3. $5x = 18$ $x =$ ______	multiply by ______ .
4. $\frac{1}{4} x = 7$ $x =$ ______	______________ . (Hint. Remember what you know about reciprocals.)

Equation	On both sides, you
5. $\frac{x}{4} = 3$ $x =$ ____	____ (Hint. This is the same as $\frac{1}{4}x = 3$.)
6. $x + {}^{-}19 = {}^{-}13$ $x =$ ____	____
7. $29x = 58$ (Simplify your answer.) $x =$ ____	____
8. $\frac{3}{4}x = 12$ $x =$ ____	____ (Reciprocal?)
9. $x + 4 = {}^{-}3$ $x =$ ____	____
10. $5x = 50$ $x =$ ____	____
11. $x + {}^{-}9 = 18$ $x =$ ____	____
12. $\frac{5}{7}x = 15$ $x =$ ____	____
13. $\frac{9}{8}x = 18$ $x =$ ____	____
14. $x + {}^{-}50 = 0$ $x =$ ____	____
15. $2x = 0$ $x =$ ____	____

Equivalent Equations

In an equation like $2x + 3 = 11$, ask yourself, "How can I get just x on the left side and a number on the right, that is: $x = ?$"

The first thing to do is to decide what was done to x to get $2x + 3$. You see that x was multiplied by 2 and then 3 was added. The result was $2x + 3$. But the same thing must have been done to the right side of the equation that stated x was a certain number ($x =$ __?__).

Class Discussion

To find the solution to $2x + 3 = 11$, you will reverse the steps that were taken to go from the equation $x = ?$ to the equation $2x + 3 = 11$. Since x was first multiplied by 2 and then 3 was added, to reverse you will first add and then multiply and you do the same thing on both sides of the equation.

Because 3 was added to both sides, add the opposite of 3 to both sides.

Left side		Right side
$2x + 3 +$ ____	$=$	$11 +$ ____

You have $2x + 0 = 8$, so you know you can write $2x = 8$. Because both sides were multiplied by 2 , multiply both sides by the reciprocal of 2 .

Left side		Right side
____ $\cdot 2x$	$=$	____ $\cdot 8$

This gives you $1 \cdot x = 4$, so you know the last equation can be written $x = 4$. You have the solution.

Sometimes, however, people make mistakes when they compute. To check, you should always go back to the first equation and replace the variable with the solution to see whether the statement is true. When you do this, you get:

$$(2 \cdot 4) + 3 = 11$$
$$8 + 3 = 11$$
$$11 = 11 .$$

This is a true statement, and you know your solution is right.

You wrote a chain of <u>equivalent</u> equations. They are equivalent because they have the same solution.

	$2x + 3 = 11$	
(and, after adding $^-3$ to both sides)	$2x = 8$	(Check your solution here.)
(and, after multiplying both sides by $\frac{1}{2}$)	$x = 4$	

In the equation, $7x + 4 = 46$, think: "First both sides were multiplied by ______ and then ______ was added. To reverse the process, I will first add the opposite of ______ to both sides and then multiply both sides by the reciprocal of ______."

<u>Left side</u>		<u>Right side</u>
$7x + 4$	$=$	46
	(Add $^-4$.)	
$7x$	$=$	42
	(Multiply by $\frac{1}{7}$.)	
x	$=$	6

Check the solution by replacing the variable, x , with 6 , in the first equation.

$$(7 \cdot 6) + 4 = 46$$

You can replace the variable with 6 in each one of the equivalent equations you wrote:

	<u>Check</u>
$7x + 4 = 46$	$(7 \cdot 6) + 4 = 46$
$7x = 42$	$7 \cdot 6 = 42$
$x = 6$	$6 = 6$

In the Exercises in Section 17-1, you had the problem $\frac{x}{4} = 3$, and you saw that this was the same as $\frac{1}{4}x = 3$. In the problem $\frac{3}{4}x = 12$, you knew that you must multiply both sides by the reciprocal of $\frac{3}{4}$. The equation $\frac{3x}{4} = 12$ has exactly the same solution as $\frac{3}{4}x = 12$, because

$$\frac{3x}{4} = \frac{3 \cdot x}{4}$$

$$= \frac{3}{4} \cdot \frac{x}{1}$$

$$= \frac{3}{4}x .$$

Remembering this helps to solve equations like $\frac{5x}{7} + 5 = 15$. Think of $\frac{5x}{7}$ as $\frac{5}{7}x$. First both sides of the equation were multiplied by ________ and then ________ was added to both sides. To reverse this process, write three equivalent equations:

Left side		Right side
$\frac{5}{7}x + 5$	=	15
	(Add ___ .)	
$\frac{5}{7}x$	=	_____
	(Multiply by ___ .)	
x	=	_____

Check your solution.

$$(\frac{5}{7} \cdot 14) + 5 = 15$$

$$10 + 5 = 15$$

$$15 = 15$$

Some equations are written in the form of subtraction.

$$3x - 2 = 10$$

Before doing anything else, change it to addition.

$$3x + {}^{-}2 = 10$$

If the variable was first multiplied by 3 and then $^{-}2$ was added, you reverse the process by adding ________ to both sides and then multiplying both sides by ________.

Left side		Right side
$3x + {}^{-}2$	=	10
	(Add ___ .)	
$3x$	=	____
	(Multiply by ___ .)	
x	=	____

Check your solution.

$$(3 \cdot 4) + {}^{-}2 = 10$$

$$12 + {}^{-}2 = 10$$

$$10 = 10$$

By now you can use the reverse process just by planning ahead and writing only the equivalent equations. With $5x + 2 = 22$, you plan first to add ________ to both sides and then to multiply both sides by ________.

$$5x + 2 = 22$$

$$5x = \_\_\_\_$$

$$x = \_\_\_\_$$

Be sure to check your solution to see if it can be used to replace x in the first equation, $5x + 2 = 22$.

$$(5 \cdot \_\_\_\_) + 2 = 22$$

$$\_\_\_\_ + 2 = 22$$

$$\_\_\_\_ = 22$$

Exercises

1. For each equation below, show your "plan" in the middle column and then write equivalent equations to find the solution. Replace the variable with your solution and check to see if the statement is true. The first one is done for you.

Equations	Plans On both sides:	Check
(a) $4x - 5 = 39$		$(4 \cdot 11) - 5 = 39$
	Rewrite as addition.	
$4x + {}^{-}5 = 39$		$44 - 5 = 39$
	Add $\underline{5}$.	
$4x = 44$		$39 = 39$
	Multiply by $\frac{1}{4}$.	
$x = 11$		
(b) $2x + 1 = 41$		________
	Add ____.	________
________	Multiply by ____.	
________		________
(c) ${}^{-}5x + 2 = 10$		________
	Add ____.	
________	Multiply by ____.	________
________		________
(d) $\frac{x}{6} - \frac{7}{2} = {}^{-}\frac{1}{3}$		________
	Rewrite as addition.	
________	Add ____.	________
________	Multiply by ____.	________
________		________
(e) $\frac{5x}{4} + 10 = 0$	Add ____.	________
________	Multiply by ____.	________
________		________

2. You are probably ready to do the planning without writing it down. For the following problems, <u>think</u> what you will have to do to both sides of the equation; then write equivalent equations. At the right, show that your solution is correct. The first one is done for you.

Equations	Check
(a) $4x - 1 = 7$	$(4 \cdot 2) + {}^{-}1 = 7$
$4x + {}^{-}1 = 7$	$8 + {}^{-}1 = 7$
$4x = 8$	$7 = 7$
$x = 2$	
(b) $2x + 10 = 50$	
(c) $4x - 15 = 7$	
(d) $2x + 5 = 5$	
(e) $\frac{2x}{5} + \frac{2}{5} = 2$	
(f) ${}^{-}3x + 10 = 1$	

Equations with a Variable in the Denominator of a Fraction

Equations sometimes have a variable in the denominator of a fraction, as in $\frac{3}{x} + 2 = \frac{7}{2}$. This looks hard, but it is solved in much the same way as the others you have done, except for some extra steps.

You know one thing about the solution of the equation before you start: x is not 0, because $\frac{3}{0}$ has no meaning.

Class Discussion

First, rewrite $\frac{3}{x}$ as $3 \cdot \frac{1}{x}$.

The equation is now:

$$3 \cdot \frac{1}{x} + 2 = \frac{7}{2}.$$

When you find out what the number $\frac{1}{x}$ is, you will know what x is, because the reciprocal of $\frac{1}{x}$ is x.

In the equation above, the number $\frac{1}{x}$ was first multiplied by 3 and then 2 was added. To solve the equation you first add _____ to both sides.

$$3 \cdot \frac{1}{x} + 2 + {}^{-}2 = \frac{7}{2} + \underline{\qquad}$$

so $$3 \cdot \frac{1}{x} = \frac{3}{2}$$

Next you multiply both sides by _____.

$$\frac{1}{3} \cdot 3 \cdot \frac{1}{x} = \underline{\qquad} \cdot \frac{3}{2}$$

so $$\frac{1}{x} = \frac{1}{2}$$

and $$x = 2$$

(If you are not sure about the last step, use the comparison property. You know that $\frac{a}{b} = \frac{c}{d}$ if $a \cdot d = b \cdot c$.)

When you multiply $1 \cdot 2$ and $1 \cdot x$ you find that $2 = x$.

Check the solution in the equation.

Equation	Check
$\frac{3}{x} + 2 = \frac{7}{2}$	$\frac{3}{2} + 2 = \frac{7}{2}$
	$\frac{3}{2} + \frac{4}{2} = \frac{7}{2}$
	$\frac{3 + 4}{2} = \frac{7}{2}$
	$\frac{7}{2} = \frac{7}{2}$

The next problem is just a little harder.

$$\frac{8}{x + 1} + 3 = 7$$

In this equation, you know that x cannot be _______ because _______ $+ 1 = 0$. Rewrite $\frac{8}{x + 1}$ as $8 \cdot \frac{1}{x + 1}$ and your equation is

$$8 \cdot \frac{1}{x + 1} + 3 = 7 .$$

First add the opposite of 3 :

$$8 \cdot \frac{1}{x + 1} = 4$$

and then multiply by the reciprocal of 8 :

$$\frac{1}{x + 1} = \frac{1}{2} .$$

Write $x + 1 = 2$ as the next step, and finally add the opposite of _______ to both sides and find the solution, $x = 1$. Now check the solution in the equation $\frac{8}{x + 1} + 3 = 7$.

Check

$$\frac{8}{1+1} + 3 = 7$$

$$\frac{8}{2} + 3 = 7$$

$$4 + 3 = 7$$

$$7 = 7$$

The equation $\frac{18}{x+7} - {}^{-}3 = 5$ gives you a chance to use many things you learned in earlier chapters. (Notice that $x \neq {}^{-}7$.)

First, rewrite the equation as addition .

$$\frac{18}{x+7} + \underline{\qquad} = 5$$

Write the equation again to show $\frac{18}{x+7}$ as 18 times a number .

$$18 \cdot \underline{\qquad\qquad} + \underline{\qquad} = 5$$

Add the opposite of ________ to both sides.

____

Multiply both sides by the reciprocal of ________.

______________________________ (Simplify the fraction on the right side.)

Because $\frac{1}{x+7} = \frac{1}{9}$, you can write:

$$\underline{\qquad\qquad\qquad} = 9 .$$

Add the opposite of ________ to both sides.

____

Check your solution by substituting it for x in the equation you started with.

$$\frac{18}{\underline{\quad} + 7} - {}^{-}3 = 5$$

$$\frac{18}{\underline{\quad}} + 3 = 5$$

$$2 + 3 = 5$$

$$5 = 5$$

Exercises

1. For each equation below, write a chain of equivalent equations to find the solution. On the right, check your solution.

Check

(a) $\frac{3}{x} - 2 = \frac{^{-}13}{8}$, and $x \neq 0$ ________

________ (Addition problem) ________

________ $(\frac{3}{x} = 3 \cdot$ ____ $)$ ________

________ (Add ___ to both sides.) ________

________ (Multiply both sides by ___.) ________

Check

(b) $\frac{2}{x+4} + \frac{1}{4} = \frac{1}{2}$, and $x \neq {}^-4$. ________________

________________ (Rewrite $\frac{2}{x+4}$ as $2 \cdot$ ____.)

________________ (Add ___ to both sides.) ________________

________________ (Multiply both sides by ___.)

________________ (If those two numbers are equal, then ________.) ________________

________________ (Add ___ to both sides.) ________________

2. In some of the problems below you will have to write only one equivalent equation. In others you will need to write two or more. Check each solution.

Check

(a) $\frac{7}{11}x = 1$ ________________

________________ ________________

(b) ${}^-5x = {}^-3$ ________________

________________ ________________

(c) $x + .29 = 6$ ________________

________________ ________________

Check

(d) $4x + 8 = 10$

(e) $\frac{3}{5}x - 4 = {}^{-}1$

(f) $\frac{x}{5} + \frac{1}{4} = \frac{17}{4}$

(g) $\frac{3}{x} + 6 = 7$, and $x \neq 0$

Solving Equations with Functions

In Chapter 8 you found that for an equation like $2x + 3 = {}^{-}1$ you could write two functions, graph both of them on a coordinate plane, and find the solution of the equation.

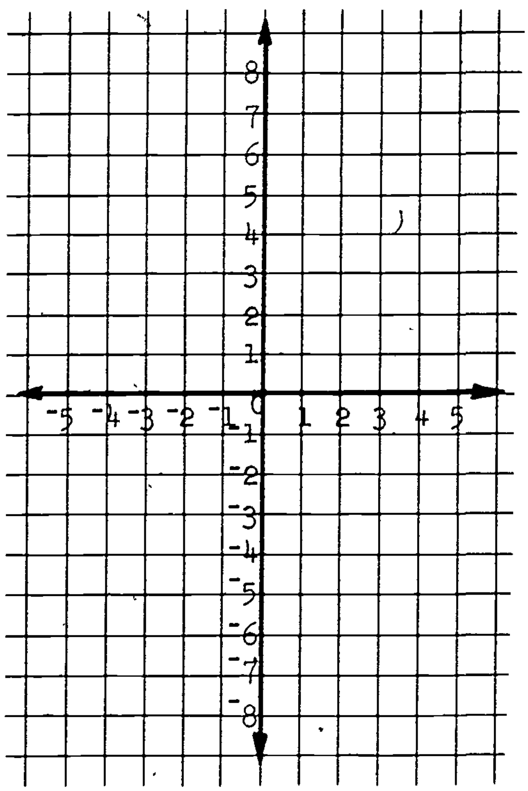
Class Discussion

To review, let's look at the equation $2x + 3 = {}^{-}1$. You can write a function using the left side: $f : x \rightarrow 2x + 3$. On the right side you can write a constant function: $g : x \rightarrow {}^{-}1$.

Very carefully graph each of these functions on the coordinate plane below. Use inputs as shown.

$f : x \rightarrow 2x + 3$	
Input	Output
1	
0	
$^{-}1$	

$g : x \rightarrow {}^{-}1$	
Input	Output
1	
0	
$^{-}1$	



The graph of the function $g : x \to {}^{-}1$ is a horizontal line, parallel to the x-axis because no matter what input you have, the output is always ______ . For what input does the graph of $f : x \to 2x + 3$ intersect the graph of $g : x \to {}^{-}1$? ______ When you replace x with ${}^{-}2$ in the equation $2x + 3 = {}^{-}1$, you have $2 \cdot {}^{-}2 + 3 = {}^{-}1$. This is a true statement, so ${}^{-}2$ is the solution to the equation.

You could also have written a chain of equivalent equations like this:

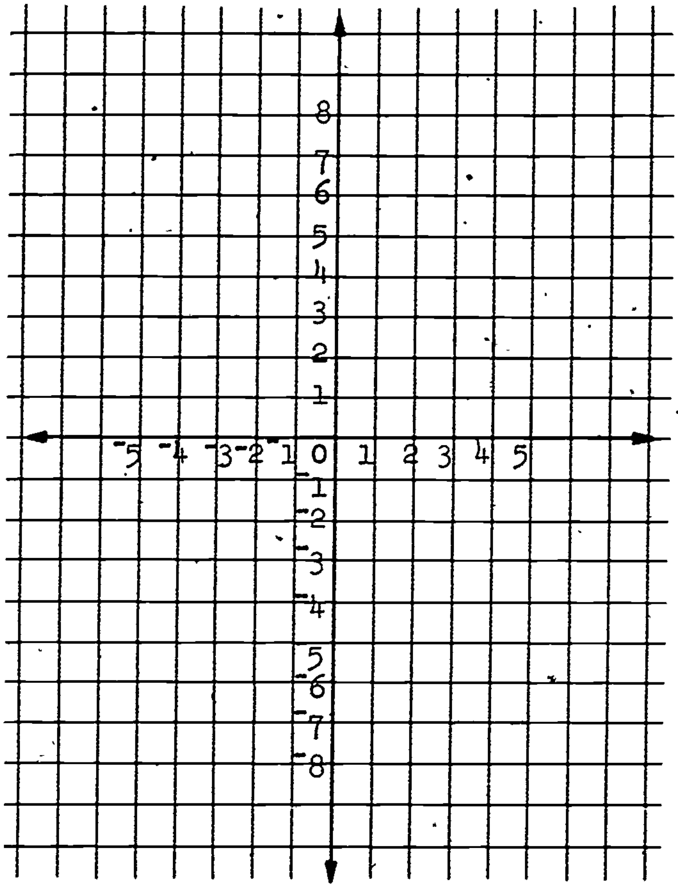
$$2x + 3 = {}^{-}1$$

$$2x = {}^{-}4$$

$$x = {}^{-}2 .$$

You can see why, in solving equations this way, it is necessary to draw the graphs very carefully and to check your solution. If you are careless, your graphs may not show the intersection correctly, and the input that gives that intersection may not be the solution of the equation.

Try the equation $4x + 6 = 2$. The function for the left side of the equation is $f : x \to$ ________ . The function for the right side of the equation is $g : x \to$ ________ . Use the inputs 0 , ${}^{-}2$, and ${}^{-}3$, and graph both functions on the next page.



What input gives the same output for both functions? ________

Find the solution of the equation $4x + 6 = 2$ by writing equivalent equations.

$$4x + 6 = 2$$

____

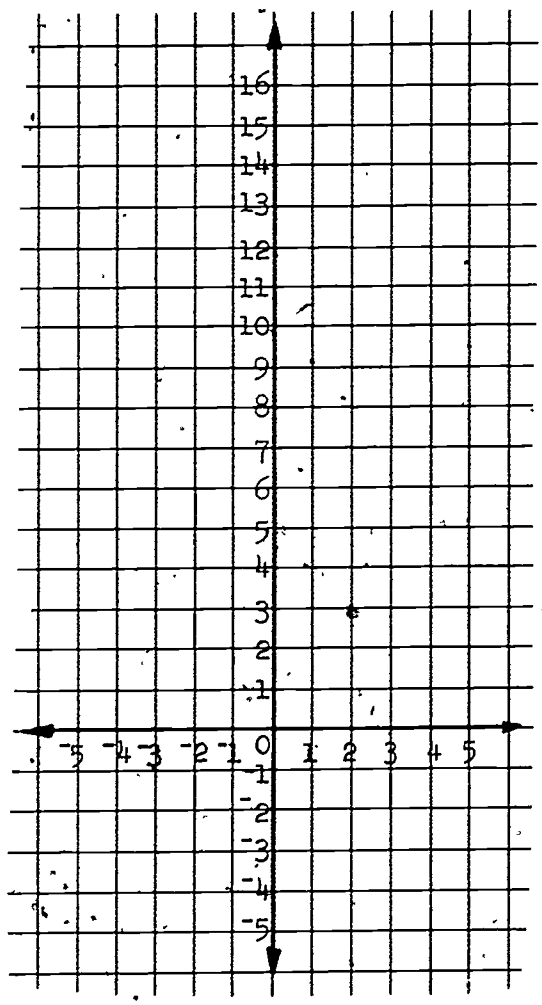
____

Did the graphs of the two functions give you this solution? ________

If not, you should try to find where you made a mistake. Check the solution in the equation, and then go back over your work.

Often the solution of an equation is not an integer, and then it is sometimes hard to see, from a graph, exactly what it is.

The equation $^-6x + 3 = 0$ is an example. The function for the left side is $f : x \rightarrow$ ________. The function for the right side is $g : x \rightarrow$ ________. This is a constant function. Use the inputs 0, $^-1$, and $^-2$ for each function and draw the graph of each.



You see that, for the equation $^-6x + 3 = 0$, the input that gives the same output for both functions is some number between 0 and 1. You might guess that it is $\frac{1}{3}$ or $\frac{1}{2}$ or $\frac{4}{5}$ or many other rational numbers. Choose one of these numbers and check it as the solution to the equation. If it is not the solution, try another of the numbers, and so on, until you find the one that solves the equation. The solution of the equation $^-6x + 3 = 0$ is ________.

The function method of solving equations does not always make it easy to see the solution of an equation exactly, and of course if you are not careful you may not find the correct one, but you should get a solution that is fairly close to the correct one. Another drawback to the function method is that the graph you use sometimes needs to be very large. In the last problem, for instance, in order to graph an input of $^-2$, the y-axis had to be at least 15 units long. How long would it have had to be to show the output for an input of $^-5$?

_______ units

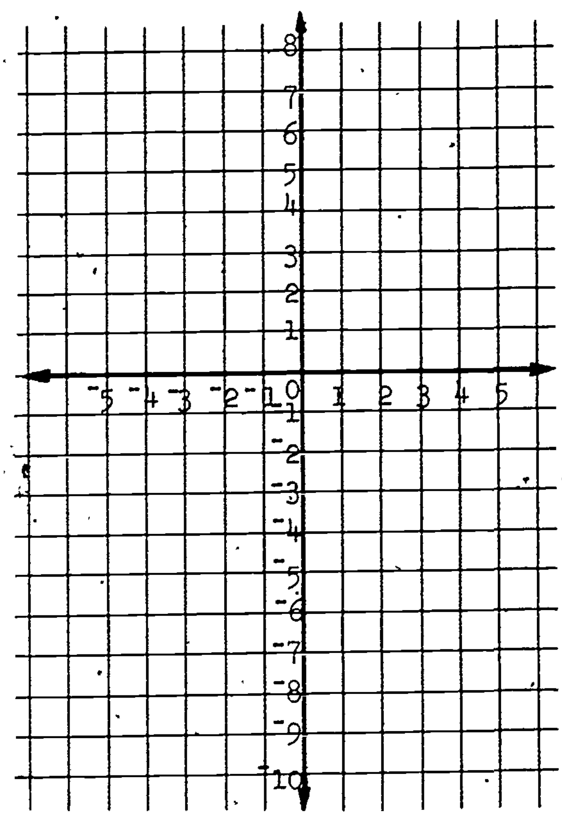
Until now, you have not been asked to solve an equation like $x + 2 = 6x + {}^-8$, in which you see x on both sides of the equation. In this kind of equation, the x on the left side of the equation <u>must</u> stand for the same number as the x on the right side. To solve this equation by the function method, write a function for the left side as usual: $f : x \rightarrow x + 2$. You can't write a constant function for the right side, but instead you can write $g : x \rightarrow 6x + {}^-8$. Use the inputs 0 , 1 , and 2 and graph these functions at the right.

What input gives the same output for both functions? _____ Check to see whether this number is the solution for the equation $x + 2 = 6x + {}^-8$. (If you were careful, it should be.)

$\_\_\_\_ + 2 = (6 \cdot \_\_\_\_) + {}^-8$

$\_\_\_\_ = \_\_\_\_ + {}^-8$

$\_\_\_\_ = \_\_\_\_$



Exercises

Write two functions for each equation. Choose inputs that allow you to graph each of the functions on the coordinate plane given. (Most of the time you have used inputs between ${}^-3$ and 3.) Answer the questions for each equation.

1. $3x + {}^-1 = {}^-4$

   (a) $f : x \rightarrow$ ____________

   (b) $g : x \rightarrow$ ____________

   (c) What input gives the same output for both functions?

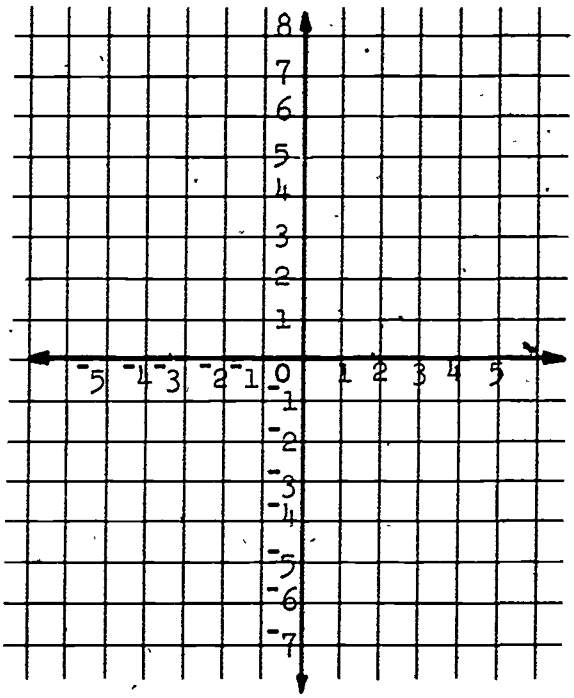
   ____

   (d) Show that this is the solution to the equation $3x + {}^-1 = {}^-4$.

   $(3 \cdot \_\_\_\_) + {}^-1 = {}^-4$

   $\_\_\_\_ + {}^-1 = {}^-4$

   ${}^-4 = {}^-4$



2. $3x + 2 = 6x + {}^-4$.

   (a) $f : x \rightarrow$ ____________

   (b) $g : x \rightarrow$ ____________

   (c) What input gives the same output for both functions?

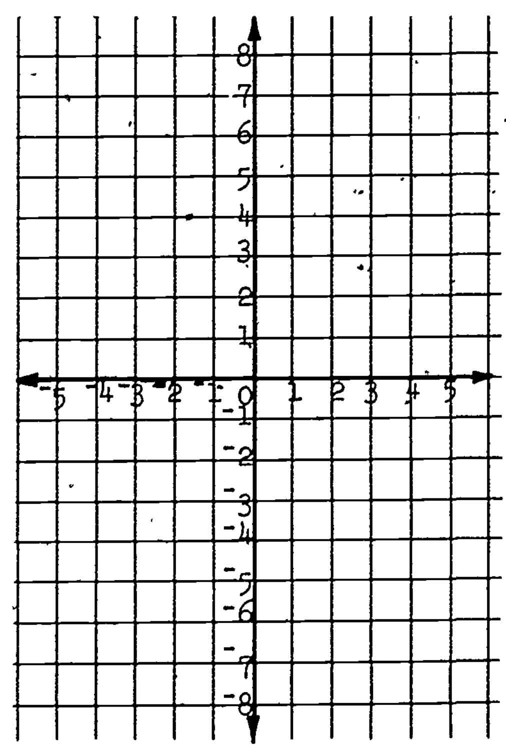
   ____

   (d) Show that this is the solution to the equation $3x + 2 = 6x + {}^-4$.

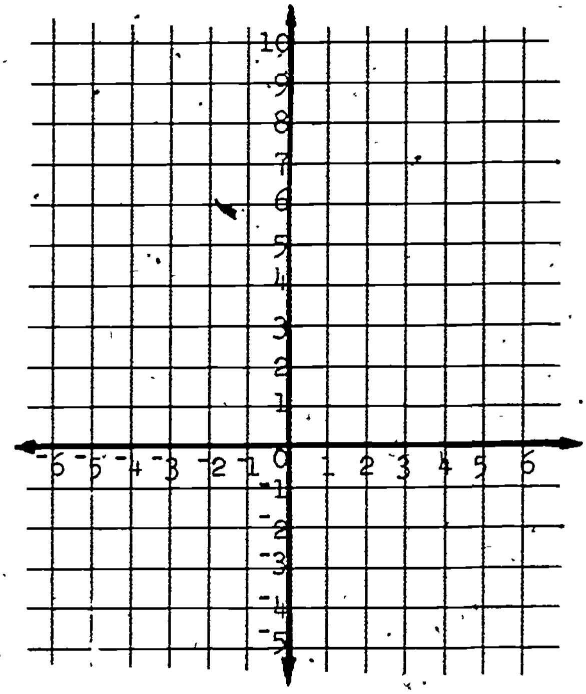
   $(3 \cdot \_\_\_\_) + 2 = (6 \cdot \_\_\_\_) + {}^-4$

   $\_\_\_\_ + 2 = \_\_\_\_ + {}^-4$

   $\_\_\_\_ = \_\_\_\_$



3. $2x + {}^-1 = x + 4$



(a) $f : x \rightarrow$ ________

(b) $g : x \rightarrow$ ________

(c) What input gives the same output for both functions?

____

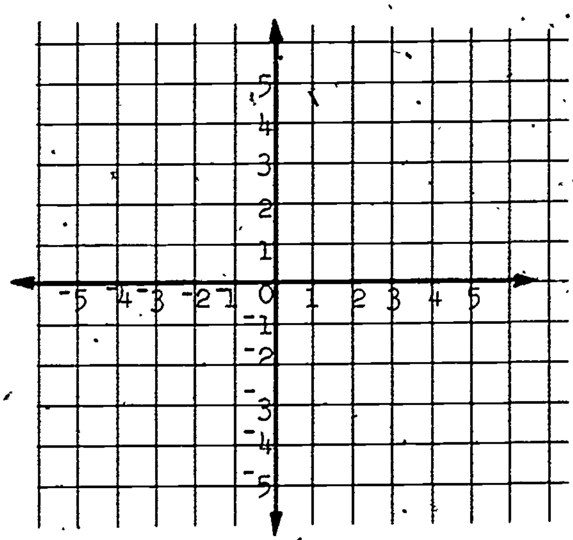
(d) Show that this is the solution to the equation $2x + {}^-1 = x + 4$.

$(2 \cdot \_\_\_\_) + {}^-1 = \_\_\_\_ + 4$

$\_\_\_\_ + {}^-1 = \_\_\_\_$

$\_\_\_\_ = \_\_\_\_$

4. $2x + {}^-1 = 1 + {}^-2x$



(a) $f : x \rightarrow$ ________

(b) $g : x \rightarrow$ ________

(c) What input gives the same output for both functions? ________ (Try to judge as well as you can.) Test your answer in the equation $2x + {}^-1 = 1 + {}^-2x$. If it is not correct, try another. Then show that you have the correct solution below.

$(2 \cdot \_\_\_\_) + {}^-1 = 1 + ({}^-2 \cdot \_\_\_\_)$

$\_\_\_\_ + {}^-1 = 1 + \_\_\_\_$

$\_\_\_\_ = \_\_\_\_$

Equations Which Have the Variable on Both Sides

Problem 4 in the last lesson is an example of an equation where the solution is not an integer and graphing does not show clearly what it is. You may also have noticed that for some equations you would need a very large piece of graph paper to find the solution by the function method. It takes only two steps more than you have used before to solve an equation like $3x + 2 = {}^{-}4 + 6x$ using equivalent equations. We will take those two steps very slowly.

Class Discussion

In solving equations you try to get, as your last equation, $x =$ some number. Up to now, the right side of the equation has not had a variable in any of the equivalent equations you wrote, so you always knew what the number on the right was.

In $3x + 2 = {}^{-}4 + 6x$, you know that ${}^{-}4 + 6x$ is the same number as $3x + 2$, but you don't know what number $3x + 2$ <u>is</u>. If you could only <u>get rid</u> of the "6x" on the right, you could use the usual method to solve the equation. When you look back at equations like $2x + 3 = 7$, you remember that you got rid of the 3 on the left side by adding __________ to both sides. To get rid of any number that has been added you add its opposite.

On the right side of $3x + 2 = {}^{-}4 + 6x$, the number $6x$ has been added to ${}^{-}4$. If you add the opposite of __________, the right side is just ${}^{-}4$. But in order to add the opposite of $6x$, you must know what the opposite <u>is</u>. By making a table you will see what happens to $6x$ and its opposite when we replace the variable with different values. Complete this table, using the values of x that are given.

Value of x	Value of $6x$	Opposite of $6x$	$^{-}6x$
5	30	$^{-}30$	$^{-}30$
4			
3			
2			
1			
0			
$^{-}1$			
$^{-}2$			
$^{-}3$			

You see that in each case the opposite of $6x$ is the same as $^{-}6x$. If you add $6x$ and $^{-}6x$, the sum is ________. Now you know that the way to get rid of the $6x$ on the right side of the equation is to add ________. But whatever you do to the right side, you must do to the left side, so you add $^{-}6x$ to the left side also. Start with $3x + 2 = {^{-}4} + 6x$, and add $^{-}6x$ to both sides:

$$3x + 2 + {^{-}6x} = {^{-}4} + 6x + {^{-}6x},$$

so $$3x + 2 + {^{-}6x} = {^{-}4}.$$

and if you add $^{-}2$ to both sides, you have

$$3x + {^{-}6x} = {^{-}6}.$$

How do you find the sum of 3 times a number and $^{-}6$ times the same number? You can substitute a few values for x to see what happens. Perhaps $3x + {^{-}6x}$ is the same as $(3 + {^{-}6})x$, or $^{-}3x$.

Value of x	$3x + {}^{-}6x$	$(3 + {}^{-}6) \cdot x$
3	$9 + {}^{-}18 = {}^{-}9$	${}^{-}3 \cdot 3 = {}^{-}9$
2	$6 + {}^{-}12 = {}^{-}6$	${}^{-}3 \cdot 2 = {}^{-}6$
1		${}^{-}3 \cdot$ ___ = ___
0		
${}^{-}1$		
${}^{-}2$		
${}^{-}3$		

You see that $3x + {}^{-}6x$ is the same as $(3 + {}^{-}6) \cdot x$, or ${}^{-}3x$. As an equivalent equation for $3x + {}^{-}6x = {}^{-}6$, therefore, you can write ${}^{-}3x = {}^{-}6$, multiply both sides by ________ and find the solution, $x =$ ________. To check the chain of equations:

	Equation	Check, substituting 2 for x
	$3x + 2 = {}^{-}4 + 6x$	$(3 \cdot 2) + 2 = {}^{-}4 + (6 \cdot 2)$
(Add ${}^{-}6x$.)	$3x + 2 + {}^{-}6x = {}^{-}4$	$6 + 2 = {}^{-}4 + 12$
(Add ${}^{-}2$.)	$3x + {}^{-}6x = {}^{-}6$	$8 = 8$
$(3x + {}^{-}6x = {}^{-}3x)$	${}^{-}3x = {}^{-}6$	
(Multiply by $\frac{{}^{-}1}{3}$.)	$x = 2$	

Exercises

1. Write the opposite of:

   (a) $3x$ ________

   (b) ^-2x ________

   (c) $\frac{1}{2}x$ ________

   (d) $\frac{^-4}{3}x$ ________

   (e) ^-10x ________

   (f) $15x$ ________

2. Add. The first one is done for you.

   (a) $4x + 3x = (4 + 3)x$

   $= 7x$

   (b) $2x + {}^-5x = (\_\_\_ + \_\_\_)x$

   $=$ ________ x

   (c) $\frac{1}{2}x + \frac{1}{2}x =$ ____________

   $=$ ________

   (d) $.5x + .7x =$ ____________

   $=$ ________

   (e) $99x + {}^-97x =$ ____________

   $=$ ________

   (f) $\frac{3}{2}x + {}^-1x =$ ____________

   $=$ ________

   (g) $^-3x + {}^-.5x =$ ____________

   $=$ ________

   (h) $1.1x + {}^-.3x =$ ____________

   $=$ ________

3. Find the solution for each equation by writing equivalent equations. At the right, check the solution in the equation. The first one is done for you.

	Equation	Check, substituting the solution for x
(a)	$3x + 5 = 2x + 4$	$(3 \cdot {}^-1) + 5 = (2 \cdot {}^-1) + 4$
(Add ${}^-2x$.)	$3x + 5 + {}^-2x = 4$	${}^-3 + 5 = {}^-2 + 4$
(Add ${}^-5$.)	$3x + {}^-2x = {}^-1$	$2 = 2$
($3x + {}^-2x = 1x$)	$x = {}^-1$	
(b)	$5x + 6 = 3x + {}^-6$	
(Add ${}^-3x$.)		
(Add ${}^-6$.)		
($5x + {}^-3x =$ ___)		
(Multiply by $\frac{1}{2}$.)		
(c)	${}^-x = 5x + {}^-3$ (Note. ${}^-x = {}^-1x$)	
(Add ${}^-5x$.)		
(${}^-x + {}^-5x =$ ___)		
(Multiply by $\frac{{}^-1}{6}$.)		

	Equation	Check, substituting the solution for x.
(d)	$3x + 8 = x + 40$	______
(Add ^-x.)	______	______
(Add $^-8$.)	______	______
($3x + {}^-x =$ ___.)	______	______
(Multiply by $\frac{1}{2}$.)	______	______
(e)	$5x + {}^-15 = x + 7$	______
(Add ^-x.)	______	______
(Add 15.)	______	______
($5x + {}^-x =$ ___.)	______	______
(Multiply by $\frac{1}{4}$.)	______ (Simplify your answer.)	______

4. Find the solution for each equation by writing equivalent equations. At the right, check the solution by substituting it for x in the equation.

	Check
(a) $4x = 3x + 105$	______
______	______
______	______

Check

(b) $4x + {}^{-}20 = 3x + 36$

(c) $2x + 1 = 3x + {}^{-}2$

Note. Sometimes it is easier to solve an equation if you simply exchange the two sides. Problem (d) shows how this works. Compare it with problem (c).

(d) $3x + {}^{-}2 = 2x + 1$

(e) $.2x + .4 = 4 + {}^{-}x$

Inequalities

An equation is a mathematical sentence that says the number on one side of the equals sign is the same as the number on the other side. It is also possible to use symbols to say that the number on the left side of the symbol is not the same as the number on the right side. One sign that can be used (like the verb in an English sentence) is $\neq$. In $3 + 4 \neq 8$, you read "3 plus 4 does not equal 8" or "3 plus 4 is not equal to 8".

Such a statement, however, does not help you compare the two numbers named, and comparing numbers is often necessary. (Which statement tells you more: "Jim is not the same age as Bill" or "Jim is younger than Bill"?)

You have used these two symbols ($<$ and $>$) for comparing two numbers. For instance, $1 + 1 < 3$ (2 is less than 3) and $3 > 1 + 1$ (3 is greater than 2). You have, perhaps, also used these symbols: $\leq$ (is less than or equal to) and $\geq$ (is greater than or equal to). The symbols $<$, $>$, $\leq$, and $\geq$ are used to express inequalities.

A statement like $4 + 5 \leq 9$ is true because $4 + 5 = 9$. If $4 + 5$ is either equal to or less than 9, the statement $4 + 5 \leq 9$ is true. Likewise, $6 + 3 \geq 8$ is true because the "is greater than" part of the symbol makes it true.

Class Discussion

1. Read each of the following sentences and tell whether it is true or false.

   (a) $4 + 5 = 9$ ________

   (b) $4 + 5 > 9$ ________

   (c) $4 + 5 \geq 9$ ________

   (d) $6 - 2 > 0$ ________

   (e) $6 - 2 \geq 0$ ________

   (f) $4 + {}^-8 < 0$ ________

   (g) $4 + {}^-4 \leq 0$ ________

   (h) $4 + {}^-8 \leq 0$ ________

   (i) $6 + 3 \geq 8$ ________

   (j) $13 - 3 \geq 10$ ________

You can use a variable in inequalities, too. First, let's think of replacing the variable only with <u>whole numbers</u>.

If $x < 1$, what number can be used to replace x? ________ If $x > 0$, you can replace x with ________________. If $x + 1 < 5$, what numbers can be used to replace x? _____, _____, _____, and _____. If you replace x with 4 then $4 + 1 < 5$ is not true. The numbers that can be used to replace the variable and make the sentence true are called the <u>solution set</u> for the sentence. (We have talked about "the solution" for equations because you have not been able to use more than one number as the solution for the equations you have had so far.)

In $x + 1 < 5$, the solution set contains the whole numbers _____, _____, _____, and _____.

2. Give the solution set (whole numbers) for each of the following statements.

(a) $x + 5 = 6$ ________

(b) $x - 8 = 1$ ________

(c) $3 + x \leq 10$ ________

(d) $x - 4 \leq 7$ ________

(e) $2 \cdot x = 18$ ________

(f) $2 \cdot x < 18$ ________

(g) $2 \cdot x \leq 18$ ________

Notice that for $2 \cdot x = 18$, you have only one number in the solution set. In $2 \cdot x < 18$, all the numbers less than 9 are in the solution set, and in $2 \cdot x \leq 18$, the number 9 <u>and</u> all of the numbers less than 9 are in the solution set.

The sentence $x + 6 > 6$ has more numbers in the solution set than anyone can possibly write down. Every whole number except 0 is included in the set, because $1 + 6 > 6$, $2 + 6 > 6$, and so on. For sentences, like this, we just give the three smallest numbers that can be used, and then write three dots to mean "and so on". For $x + 6 > 6$, we write $1, 2, 3, \ldots$ This shows that ______ is not in the set but the rest of the whole numbers are.

Sometimes there is <u>no</u> number you can use to replace the variable and make a true statement. For instance, if we use only the whole numbers, there is no number that makes $x + 6 < 6$ true. In this case, we say that the solution set is <u>empty</u>. In the equation $2x + 3 = 2x + 5$, there is no replacement for x that makes the statement true. If we add $^{-}2x$ to both sides, we have $3 = 5$, which certainly is not true. So the solution for the equation $2x + 3 = 2x + 5$ is <u>the empty set</u>. The symbol for the empty set is $\{ \ \}$.

Exercises

Using only whole numbers for x, give the solution set for each of the following.

1. $x < 3$ ________________
2. $x + 2 < 3$ ________________
3. $4 + x > 13$ ________________
4. $5 + 2 < x$ ________________
5. $7 + x \geq 29$ ________________
6. $x + 1 < x$ ________________
7. $2x < 25$ ________________
8. $2x + 1 < 25$ ________________
9. $2x < 4$ ________________
10. $5x > 10$ ________________
11. $4x < 12$ ________________
12. $\frac{x}{2} < 8$ ________________
13. $\frac{x}{3} > 5$ ________________
14. $3x \leq 15$ ________________
15. $3x + 1 \leq 15$ ________________
16. $3x + 2 \leq 15$ ________________
17. $3x + 3 \leq 15$ ________________
18. $3x + 3 < 15$ ________________
19. $15x > 15$ ________________
20. $2x + 1 < 1$ ________________

Solving Inequalities with Graphs

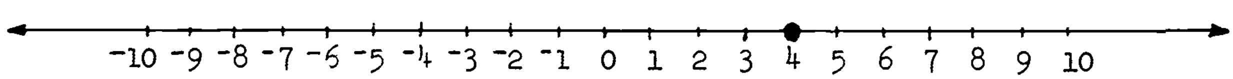
In the last section, you solved inequalities using whole numbers. One way to solve an inequality is just to guess a solution, try it, and then guess some others until you find which numbers fit. This works with the kinds of problems you just had, where the answers are all whole numbers, although you can imagine that just guessing would not be a very efficient way to solve some inequalities. Furthermore, if we include the integers, rational numbers, or real numbers as possible numbers in the solution set, it would be impossible to write solution sets in the form you just used.

First, let's find a way to show the solution set on the number line.

Class Discussion

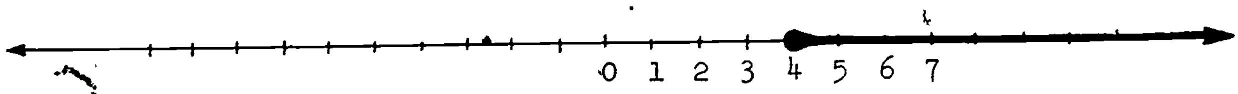
What is the solution of the equation $2x + 3 = 11$? $x =$ ______.

To show this on the number line, you just make a large dot at the point that corresponds to 4 .



In the inequality, $2x + 3 \geq 11$, any number greater than 4 will also make the statement true.

If you want to show $2x + 3 \geq 11$ on the number line, you draw a <u>ray</u> with its endpoint at 4 and pointing to the right.

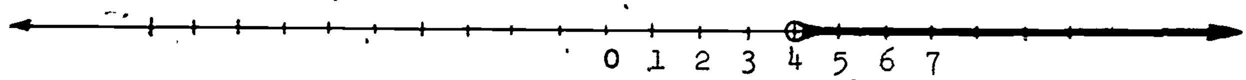


Or you can write the solution set using symbols, like this:

$$x \geq 4.$$

This means that you can replace x in $2x + 3 \geq 11$ with 4 or any number greater than 4 (for instance, with 7 , or $\frac{17}{4}$, or $\sqrt{27}$) and have a true statement.

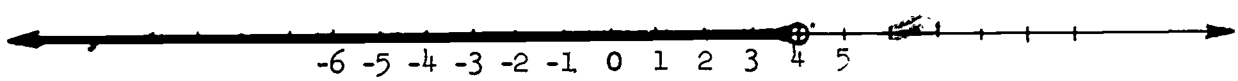
If the inequality is $2x + 3 > 11$, you must make it clear that 4 is <u>not</u> in the solution set, but that all numbers greater than 4 <u>are</u>. On the number line, the fact that 4 is not in the set is shown by the small circle at that point.



Using symbols, you can write:

$$x > 4 .$$

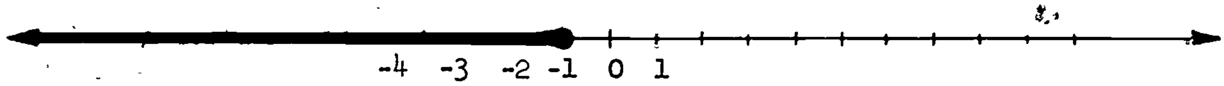
To show $2x + 3 < 11$, you show that 4 is not in the set but all numbers less than 4 are.



Using symbols, you can write: x ____________.

The number line below shows the solution set for an inequality. Write the symbol that goes in the blank.

x + 1 _____ 0

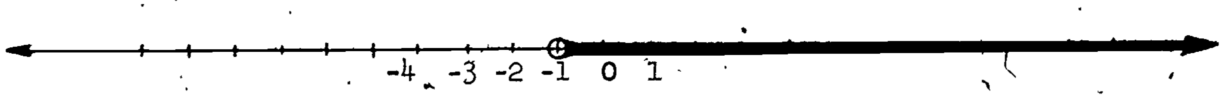


Write the solution set using symbols.

x __________

Write the symbol that goes in the blank of this inequality. The solution set is shown on the number line below.

x + 1 _____ 0

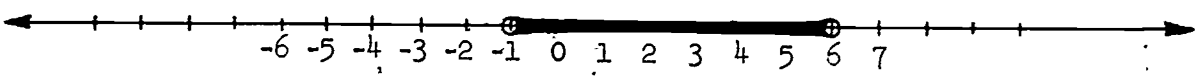


Write the solution set. x ___________

You will sometimes see statements like this:

$$3 < x + 4 < 10 .$$

This means that 3 is less than $x + 4$ and $x + 4$ is less than 10. An easier way to say it is that $x + 4$ is <u>between</u> 3 and 10, but in symbols, it is written as it is above. To find the solution set, we can think: "If $x = {}^{-}1$, then $x + 4 = 3$, so ${}^{-}1$ is not in the set, and if x is 6, then $x + 4 = 10$, so 6 is not in the set." All the numbers between ${}^{-}1$ and 6 <u>are</u> in the solution set. To show this on the number line we draw a segment with the endpoints, ${}^{-}1$ and 6, circled.

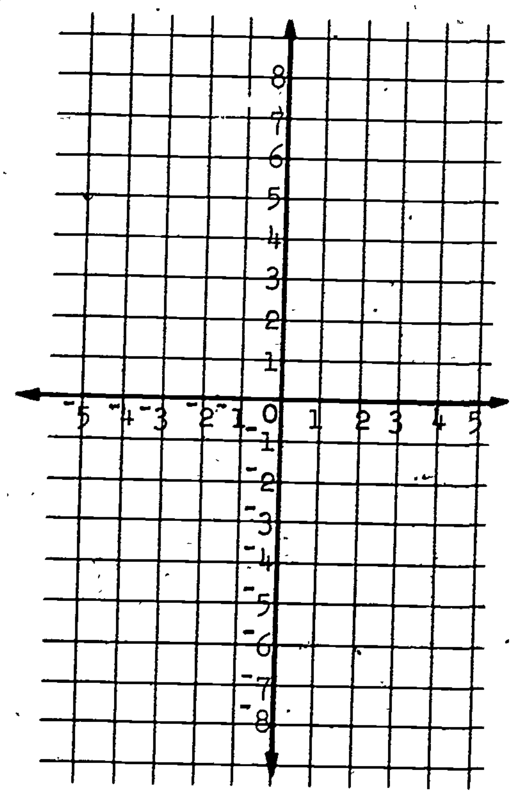


To show every number <u>between</u> ${}^{-}1$ and 6, the solution set is written: ${}^{-}1 < x < 6$.

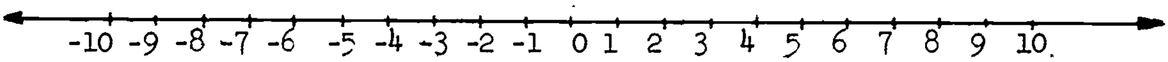
One way to solve inequalities is to graph them. Let's start with $3 \leq x + 4$. First, knowing that 3 may <u>equal</u> $x + 4$, write two functions for the equation $3 = x + 4$.

$f : x \rightarrow$ ____________ and

$g : x \rightarrow$ ____________ . Graph these two functions on the coordinate plane at the right.



The input that gives the same output for both functions is _____. So, if $3 = x + 4$, the solution is $^{-}1$. But suppose $3 < x + 4$. You know the solution set contains more than one number, and you can use just a few tests to find out what they are. Pick a number that is less than $^{-}1$ and try it in the inequality: $3 <$ _____ $+ 4$. Is the statement true or false? ________ Try a number that is greater than $^{-}1$. $3 <$ _____ $+ 4$. Is this true or false? ________ Now you are ready to show the solution set on the number line. Since $^{-}1$ is in the set, mark the point $^{-}1$ with a large dot. Any number greater than $^{-}1$ is in the set, so the ray will go to the __________. Show the solution set of $3 \leq x + 4$ on the number line below.



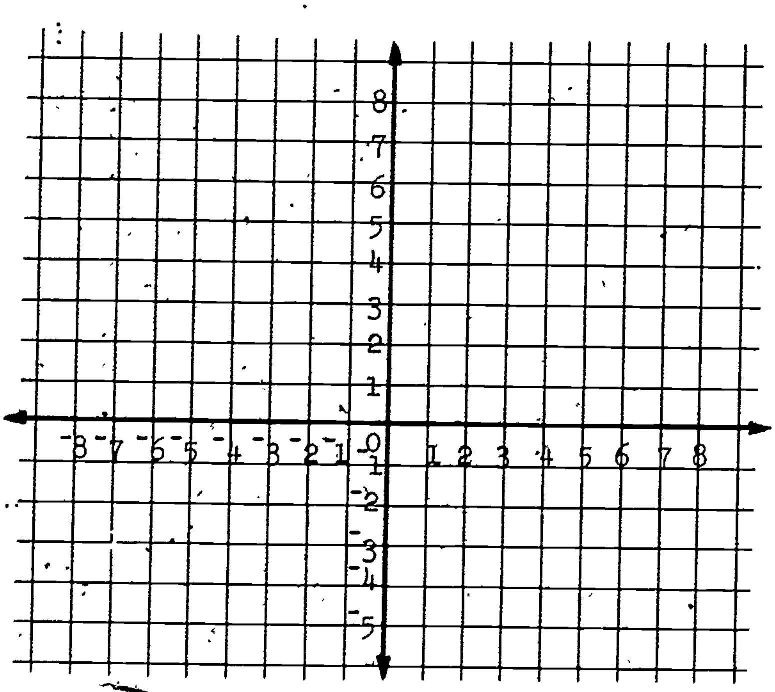
Write the solution set using symbols. x____________

You can use the same method to solve inequalities using just $<$ or $>$ symbols.

To solve $x + 6 < 1$, first write an <u>equation</u>: $x + 6 = 1$. Write two functions for the equation, and graph the solution.

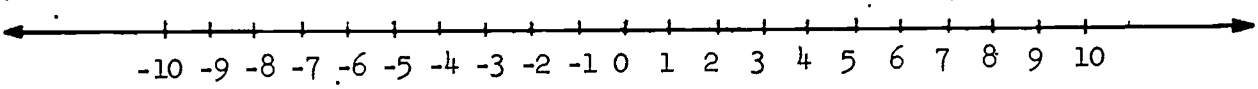
$f : x \rightarrow$ ________

$g : x \rightarrow$ ________



The solution of the <u>equation</u> $x + 6 = 1$ is ______. Show this on the number line as a circle at that point, because $^-5$ is <u>not</u> in the solution set of $x + 6 < 1$.

Now try a number to the right of $^-5$ in the inequality. Are numbers to the right of $^-5$ in the solution set? ________ Try a number to the left of $^-5$. Is it in the solution set? ________ The ray that begins at the circle should point to the ________. Show the solution set on the number line below.

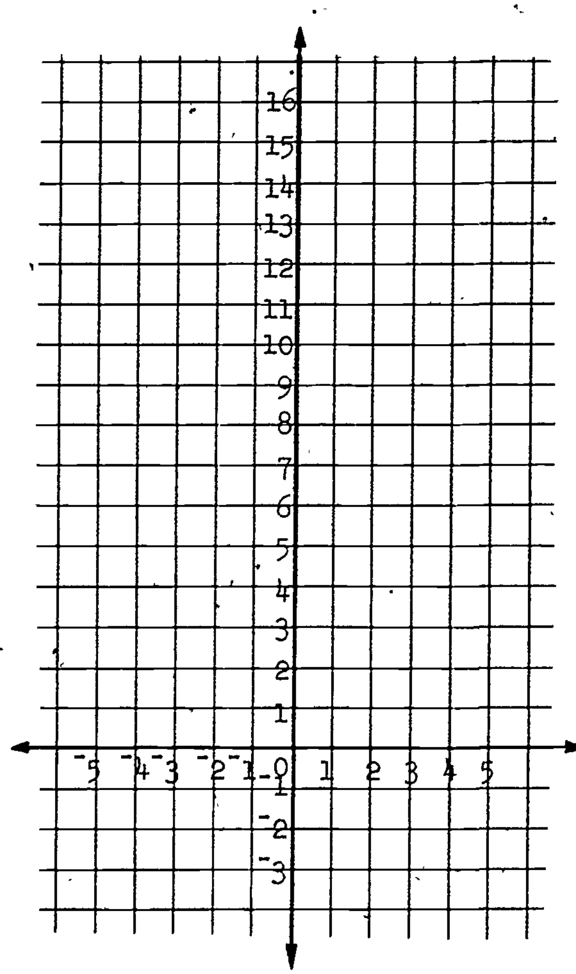


Write the solution set using symbols. x__________

Some inequalities have variables on both sides of the inequality symbol. To solve $3x + 4 < {}^-2x + 14$, do just what you did before. First, replace the $<$ with $=$. This gives you $3x + 4 = {}^-2x + 14$, an equation. You know how to solve this by graphing two functions. (Use inputs $^-1$, 0, and 1.)

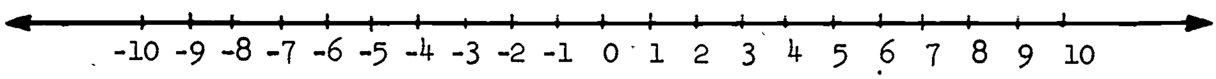
$f : x \rightarrow 3x + 4$

$g : x \rightarrow {}^-2x + 14$



Write the solution of the equation $3x + 4 = {}^{-}2x + 14$.

$x =$ ________. Try a number greater than the solution of the equation as a replacement for x in $3x + 4 < {}^{-}2x + 14$. Does it make the statement true? ________ Try a number less than the solution of the equation. Does it make the statement true? ________ Show the solution set on the number line below.



Write the solution set using symbols. x ____________

Exercises

Solve these inequalities by writing an equation and graphing two functions. Find the solution set and show it on the number line.

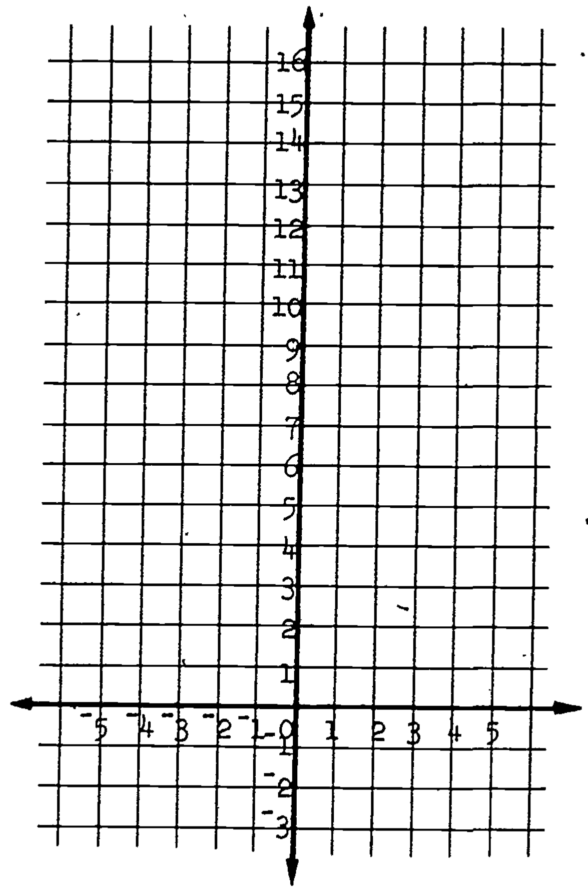
1. $13 + {}^{-}2x \geq 5$

____

(Equation)

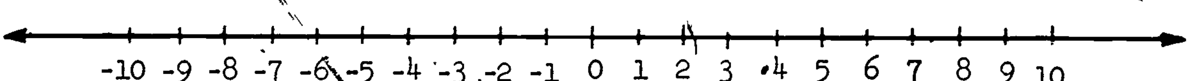
$f : x \rightarrow$ _______________ .

$g : x \rightarrow$ _______________



Solution of equation:

$x =$ _______________



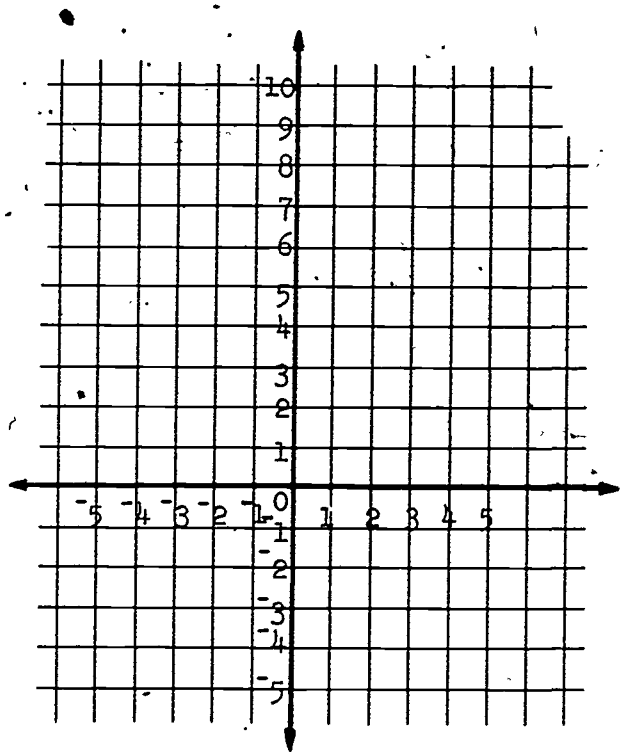
Solution set of $13 + {}^{-}2x \geq 5$: _______________ .

2. $3x + {}^-2 < {}^-x + 10$

____
(Equation)

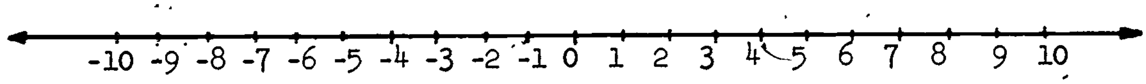
$f : x \rightarrow$ ________

$g : x \rightarrow$ ________



Solution of equation:

$x =$ ____________________



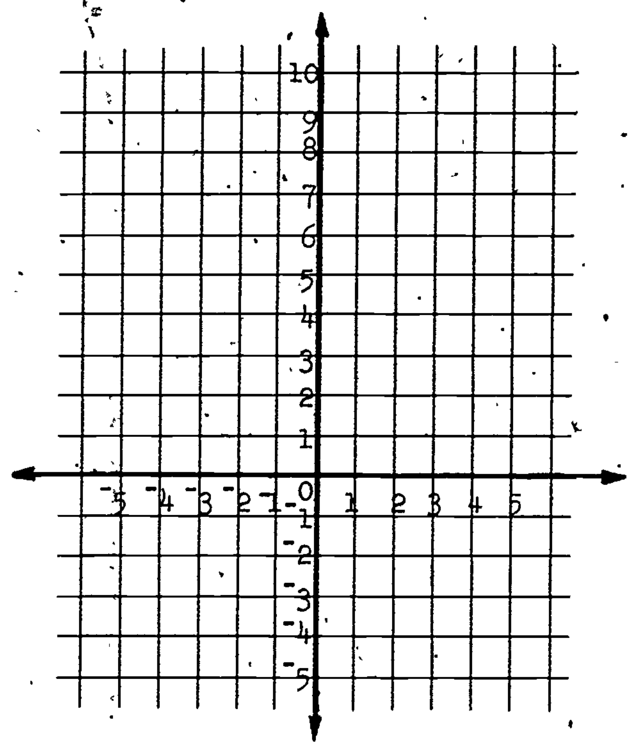
Solution set of $3x + {}^-2 < {}^-x + 10$: ____________________

3. $5x + {}^{-}2 < 2x + 1$

____
(Equation)

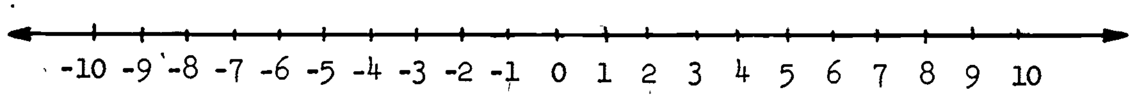
$f : x \rightarrow$ ________

$g : x \rightarrow$ ________



Solution of equation:

$x =$ ______________________



Solution set of $5x + {}^{-}2 < 2x + 1$: ______________________

Equivalent Inequalities

Just as you can solve an equation by writing a chain of equivalent equations, you can solve inequalities by writing a chain of equivalent inequalities. There is, however, one slight difference, which you will see if you work with some inequalities without variables.

Class Discussion

Put < or > in the blank to make each a true statement.

1. 3 ___ 5

   $3 + 4$ ___ $5 + 4$

2. $^{-}9$ ___ 5

   $^{-}9 + {}^{-}3$ ___ $5 + {}^{-}3$

3. 13 ___ 7

   $13 + 8$ ___ $7 + 8$

4. 4 ___ $^{-}1$

   $4 + 6$ ___ $^{-}1 + 6$

If you add the same number to both sides of an inequality, the inequality symbol does not change.

Put the correct symbol in the blank to make each a true statement. Notice how the problems on the left are different from those on the right.

5. 2 ___ 7

   $3 \cdot 2$ ___ $3 \cdot 7$

6. $^{-}3$ ___ 2

   $4 \cdot {}^{-}3$ ___ $4 \cdot 2$

7. 8 ___ $^{-}1$

   $2 \cdot 8$ ___ $2 \cdot {}^{-}1$

8. $^{-}5$ ___ $^{-}6$

   $1 \cdot {}^{-}5$ ___ $1 \cdot {}^{-}6$

9. 2 ___ 7

   $^{-}3 \cdot 2$ ___ $^{-}3 \cdot 7$

10. $^{-}3$ ___ 2

    $^{-}4 \cdot {}^{-}3$ ___ $^{-}4 \cdot 2$

11. 8 ___ $^{-}1$

    $^{-}2 \cdot 8$ ___ $^{-}2 \cdot {}^{-}1$

12. $^{-}5$ ___ $^{-}6$

    $^{-}1 \cdot {}^{-}5$ ___ $^{-}1 \cdot {}^{-}6$

When you multiplied both sides of the inequality by a <u>positive</u> number, the sign did not change. When you multiplied both sides by a <u>negative</u> number, the sign had to be reversed. This fact is important in solving inequalities in which you must multiply by a negative number.

You can solve many inequalities by adding the same number to both sides. To solve $x + 3 < 5$, you add _____ to both sides to get $x + 0 <$ ___ or simply $x <$ ___. $x + 3 < 5$ and $x < 2$ are <u>equivalent inequalities</u> because they have the same solution set.

To solve $x + {}^{-}5 > 8$ you add _____ to both sides in order to get $x + 0 >$ _____ or $x >$ _____.

13. Solve each of these inequalities by adding the same number to both sides.

(a) $x + 9 < 3$

$x <$ __________

(b) $x - 8 < 21$

__________ (Rewrite as addition.)

$x <$ __________

(c) $x + {}^{-}10 > 6$

$x >$ __________

(d) $x - {}^{-}2 < 14$

____

$x <$ __________

(e) $x + 7 \leq 18$

$x \leq$ __________

(f) $x + {}^{-}9 \geq {}^{-}15$

$x \geq$ __________

Sometimes you must multiply both sides of an inequality by the same number to find the solution set. In $4x < 12$, you multiply both sides by the reciprocal of ______. The solution is, $x <$ ______. In $\frac{4}{5}x > 20$, you multiply both sides by ______. $x >$ ______.

Look at this one: $^{-}4x < 12$. If you just multiply both sides by the reciprocal of $^{-}4$, which is $\frac{^{-}1}{4}$, you get $x < {}^{-}3$. One number in the solution set of $x < {}^{-}3$ is $^{-}4$. Check to see whether it is also in the solution set of $^{-}4x < 12$.

$$^{-}4 \cdot {}^{-}4 = 16$$

and 16 is <u>not</u> less than 12.

> Remember that if you multiply both sides of an inequality by a <u>negative</u> number, you must reverse the inequality symbol.

If you multiply $\frac{^{-}1}{4} \cdot {}^{-}4x$ and $\frac{^{-}1}{4} \cdot 12$, you must write, as the solution set, $x > {}^{-}3$. Try a number greater than $^{-}3$ in the inequality $^{-}4x < 12$. If you try $^{-}2$, you find that $^{-}4 \cdot {}^{-}2$ is 8, and that <u>is</u> less than 12. In order to keep from making mistakes, <u>always</u> check your answer by trying a number in the solution set to see whether it fits in the inequality you tried to solve.

14. Solve each of these inequalities by multiplying both sides by the same number. Be careful when you multiply by a negative number!

(a) $8x \geq \frac{1}{2}$

$x \geq$ ______

(b) $\frac{1}{3}x < 7$

$x <$ ______

(c) $\frac{4}{3}x \leq 12$

$x \leq$ ______

(d) $^{-}3x > 15$

x ______ ______

(What sign must you have?)

Last, you can use equivalent inequalities to solve problems which have the variable on both sides, just as you solved equations.

To solve $5x + {}^-2 < 2x + 1$, you first add ${}^-2x$ to both sides:

$$5x + {}^-2 + {}^-2x < 1 .$$

and then add 2 to both sides:

$$5x + {}^-2x < 3 .$$

Finally, you write $5x + {}^-2x$ as $3x$:

$$3x < 3$$

and multiply both sides by $\frac{1}{3}$:

$$x < 1 .$$

To check, use a number that is in the solution set of $x < 1$ to see whether it is also in the solution set of the inequality you started with. Zero is in the solution set, and if you use 0 , you see:

$$5 \cdot 0 + {}^-2 < 2 \cdot 0 + 1$$

$$^-2 < 1 .$$

This is true, and the solution set checks.

Exercises

Solve these inequalities by writing equivalent inequalities. At the right, use a number from the solution set to show that the solution set is correct.

Check

1. $6x + 3 > 7 + 5x$ ______________

______________ (Add $^{-}5x$.) ______________

______________ (Add $^{-}3$.) ______________

______________ $(6x + {}^{-}5x = ?)$ ______________

2. $5x + 11 < 3x + 3$ ______________

______________ ______________

______________ ______________

______________ ______________

______________ ______________

3. $100x + {}^{-}14 < 99x + {}^{-}10$ ______________

______________ ______________

______________ ______________

______________ ______________

4. $3x + {}^{-}2 < {}^{-}x + 10$

5. ${}^{-}2x + 15 < x + {}^{-}12$

Solving Quadratic Equations

You have used the word "squared" when you raised a number to the second power. 10^2 may be read "10 to the second power" or "10 squared". The same idea can be used in writing $x \cdot x$ as x^2. This is read "x to the second power" or "x-squared". Equations like $x^2 - 2x - 3 = 0$ are called <u>quadratic</u> equations. The word "quadratic" comes from a Latin word meaning to make something square. In this section, you will learn one way to solve quadratic equations.

Class Discussion

If you have a very simple equation, like $x^2 = 4$, it is easy to see the solution set. What numbers can you substitute for x ?
________ and ________ We often call the numbers 2 and $^-2$ the <u>roots</u> of the equation $x^2 = 4$.

Of course, not all equations have rational roots. The equation $x^2 = 3$, for instance, has two real numbers in the solution set, $\sqrt{3}$ and $^-\sqrt{3}$, but it has no rational roots. In this section you will solve only equations whose solution sets are rational numbers.

The equation $x^2 + {}^-4 = 5$ can be solved by using what you learned about getting the variable by itself on one side of the equal sign and everything else on the other side. If you want to get rid of $^-4$ on the left side, you add ________ to both sides of the equation.

$$x^2 + {}^-4 + \_\_\_\_ = 5 + \_\_\_\_$$

so $$x^2 = \_\_\_\_$$

and $$x = \_\_\_\_ \text{ or } x = \_\_\_\_$$

To solve $2x^2 = 2$, you multiply both sides by ________.

$$\_\_\_\_ \cdot 2x^2 = \_\_\_\_ \cdot 2$$

so $$x^2 = \_\_\_\_$$

and $$x = \_\_\_\_ \text{ or } x = \_\_\_\_$$

Exercises

Solve the following equations. Check both of your solutions in the equation. The first one is done for you.

Check

1. $\frac{2}{3}x^2 = 6$ Both $3 \cdot 3$ and $^-3 \cdot {}^-3 = 9$, so

$x^2 = \underline{\ 9\ }$ $\frac{2}{3} \cdot \underline{\ 9\ } = \underline{\ 6\ }$

$x = \underline{\ 3\ }$ or $x = \underline{\ ^-3\ }$ $\underline{\ 6\ } = \underline{\ 6\ }$

2. $\frac{x^2}{2} = 8$ (Remember this is the same as $\frac{1}{2}x^2 = 8$.)

$x^2 = \_\_\_\_$ ____________

$x = \_\_\_\_$ or $x = \_\_\_\_$ ____________

3. $4x^2 + {}^-3 = 97$

$4x^2 = \_\_\_\_$ ____________

$x^2 = \_\_\_\_$ ____________

$x = \_\_\_\_$ or $x = \_\_\_\_$ ____________

4. $x^2 + {}^-17 = 32$

$x^2 = \_\_\_\_$ ____________

$x = \_\_\_\_$ or $x = \_\_\_\_$ ____________

5. $3x^2 + 6 = 18$

$3x^2 = \_\_\_\_$ ____________

$x^2 = \_\_\_\_$ ____________

$x = \_\_\_\_$ or $x = \_\_\_\_$ ____________

Class Discussion

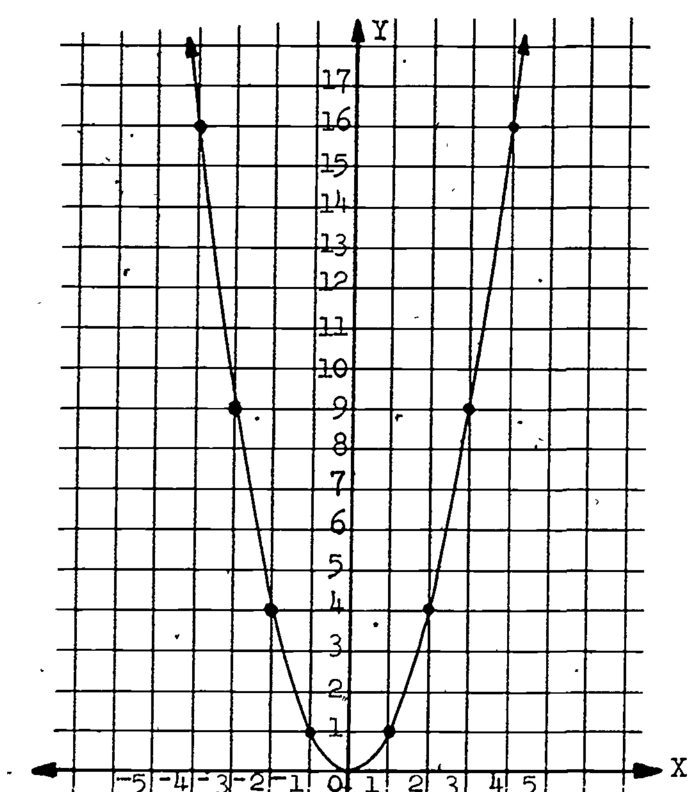
You may be able to guess the solution set of this equation: $x^2 = 2x$, but we can use it to show one way to solve harder ones. Again we will write two functions. From the left side of the equation, we write the function $f : x \to x^2$. From the right side we write $g : x \to 2x$. Fill in the table of inputs and outputs for $f : x \to x^2$.

Input x	Output x^2
$^{-}3$	9
$^{-}2$	
$^{-}1$	
0	
1	
2	
3	

The graph of the function $f : x \to x^2$ is shown below. <u>This is called the basic quadratic function</u>, and, as you see, it is not a line but a <u>curve</u>. Notice that the bottom point of the curve is at 0. For the function $f : x \to x^2$, is there any input that would give a negative output? ______ When you multiply either a positive or a negative number by itself, the product is always ______.

(Positive or negative)

Using the graph in the figure at the right, draw the graph of the function $g : x \to 2x$, which <u>is</u> a line.



$f : x \to x^2$

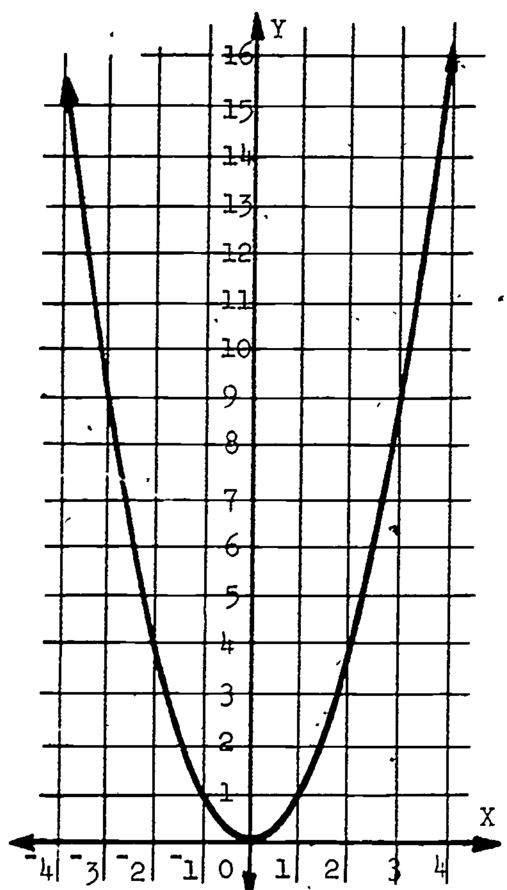
In how many points does the graph of $g : x \to 2x$ intersect the curve of $f : x \to x^2$? ________ What inputs give the same outputs for both functions? ________ and ________ The solution set of the equation $x^2 = 2x$ is: $x =$ _____ or $x =$ _____ . Check these solutions in the equation $x^2 = 2x$.

$$(\_\_ \cdot \_\_) = (2 \cdot \_\_)$$

and $$(\_\_ \cdot \_\_) = (2 \cdot \_\_)$$

To solve the equation $x^2 = 2x + 3$, graph two functions: $f : x \to$ ______________ and $g : x \to$ ______________ .

The graph of $f : x \to x^2$ is drawn for you in the figure below. Draw the graph of the function g and find what inputs give the same outputs for both functions. $x =$ _____ or $x =$ _____ .



$f : x \longrightarrow x^2$

When you draw the graph of the function $g : x \to 2x + 3$ very carefully, you find that it intersects the graph of the function $f : x \to x^2$ at the points for the input 3 and the input $^-1$. To check these inputs as solutions for the equation $x^2 = 2x + 3$:

$$(3 \cdot 3) = (2 \cdot 3) + 3 \quad \text{and} \quad (^-1 \cdot {}^-1) = (2 \cdot {}^-1) + 3$$

$$9 = 6 + 3 \qquad\qquad 1 = {}^-2 + 3$$

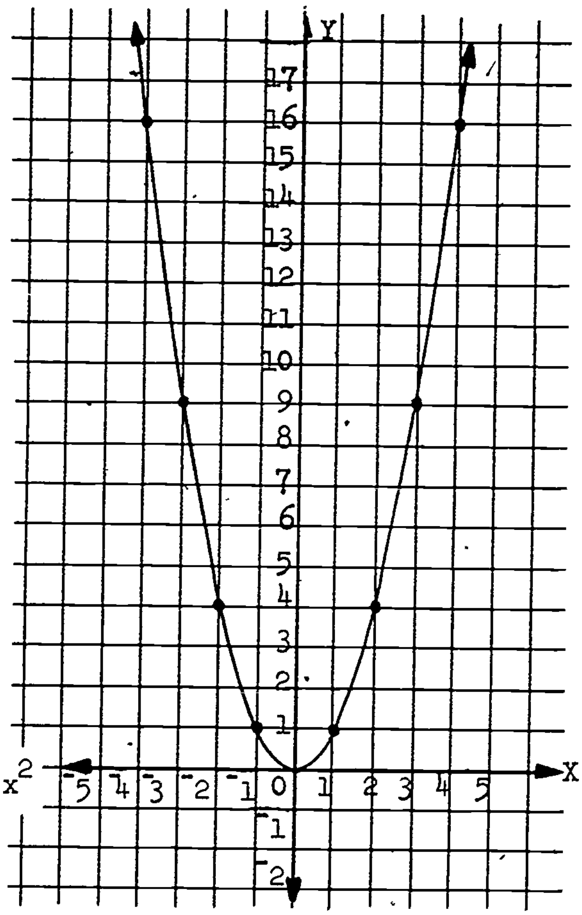
$$9 = 9 \qquad\qquad 1 = 1$$

Going back to the equation at the beginning of this section, $x^2 - 2x - 3 = 0$, you can now find a way to solve it. You know how to solve an equation if you have only x^2 on the left side, so the problem is to get rid of the rest of what's on the left side. First, rewrite the subtraction expressions $- 2x$ and $- 3$ as addition: $+ {}^-2x$ and $+ {}^-3$. The equation now is $x^2 + {}^-2x + {}^-3 = 0$. To get rid of ^-2x , you ______________ its opposite to both sides, and to get rid of $^-3$ you also add its ______________ to both sides. You don't even need two steps for this. You find that $x^2 = 2x + 3$. This is the same equation you solved before! So the solution for the equation $x^2 - 2x - 3 = 0$ is $x =$ _____ or $x =$ _____ . Check these solutions in the equation.

Look at the equation $x^2 + 3x + 2 = 0$. First you want to get rid of the $3x$ and the 2 that were added to x^2 . Add __________ and __________ to both sides of the equation to get an equivalent equation:

$$x^2 = \underline{\hspace{4cm}} .$$

The function $f : x \rightarrow x^2$ is graphed below. Graph the function $g : x \rightarrow$ __________ that goes with the right side of the equation.



$f{:}x \rightarrow x^2$

You may find it hard to read your graph for this equation, but you can see that the two integer inputs which give the same output for both functions are ______ and ______. Check these as solutions of the equation $x^2 + 3x + 2 = 0$.

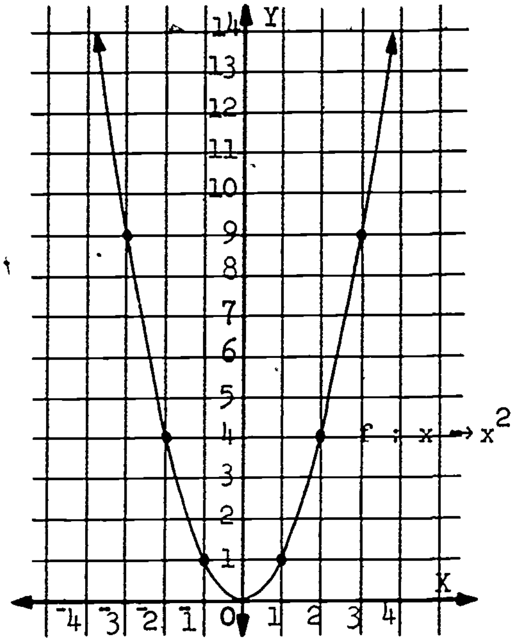
(___ · ___) + (3 · ___) + 2 = 0 and (___ · ___) + (3 · ___) + 2 = 0

___ + ___ + 2 = 0   ___ + ___ + 2 = 0

___ = 0   ___ = 0

Exercises

Solve the following equations as you did the ones above. Then check each solution in the equation.



1. $x^2 - x - 6 = 0$

_______________ (Rewrite as addition.)

$x^2 =$ _______ (Add ____ and ____.)

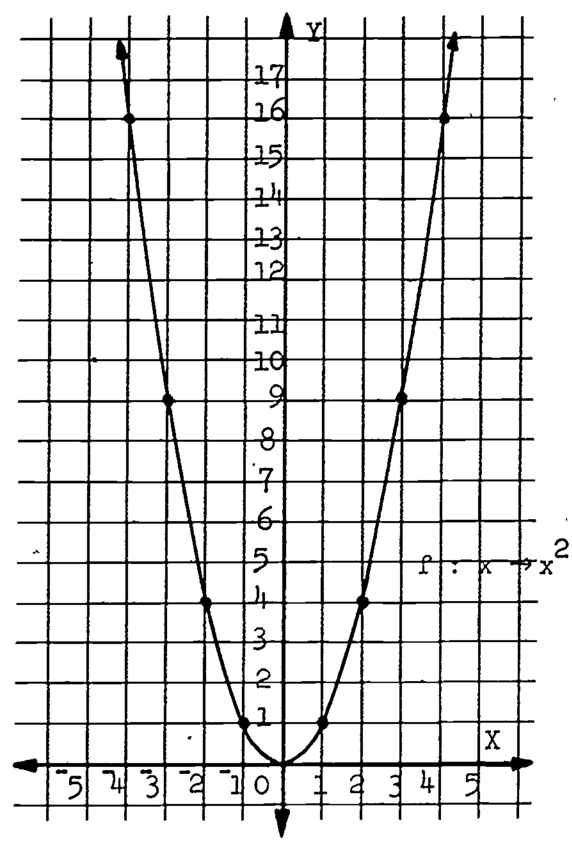
$f : x \rightarrow$ _______________

$g : x \rightarrow$ _______________

$x =$ ______ or $x =$ ______

Check:

_______________ and _______________



2. $x^2 - 3x - 4 = 0$

_______________ (Rewrite as addition.)

$x^2 =$ _______

$f : x \rightarrow$ _______________

$g : x \rightarrow$ _______________

$x =$ ______ or $x =$ ______

Check:

_______________ and _______________

BRAINBOOSTER.

3. (a) You have solved equations in which you had just x^2 on the left side of the equals sign. For the equation below, you want to get $2x^2$ on the left side.

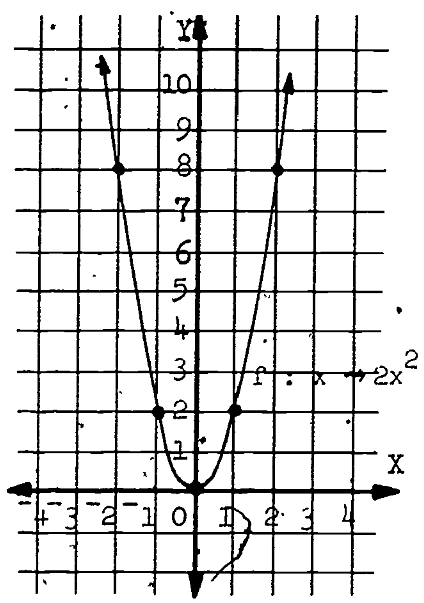
$2x^2 - 5x + 2 = 0$

________________ (Rewrite as addition.)

$2x^2 =$ ____________ (Add _____ and _____.)

(b) The function for the left side of the equation you now have is $f : x \rightarrow$ __________.

(c) The function for the right side of the equation is $g : x \rightarrow$ __________.
The graph of the function $f : x \rightarrow 2x^2$ is drawn for you. Graph the function g.



(d) One input that gives the same output for both functions is the integer _____. Check this integer solution in the equation.

$2 \cdot (\_\_ \cdot \_\_) - (5 \cdot \_\_) + 2 = 0$

(e) The other input that gives the same output is not an integer but a rational number between ______ and ______. Make a guess as to what the number is and try your solution in the equation. If you don't guess the right one the first time, try another rational number. In the equation

$2x^2 - 5x + 2 = 0$, $x =$ _____ or $x =$ _____.

Pre-Test Exercises

These exercises are like the problems you will have on the chapter test. If you don't know how to do them, read the section again. If you still don't understand, ask your teacher.

1. (Section 17-1.)
For each equation, write what to do to both sides of the equation in order to get x by itself on the left side of the equal sign.

(a) $x + 4 = 9$ ____________________

(b) $x + {}^{-}3 = 17$ ____________________

(c) $4x = 16$ ____________________

(d) $\frac{{}^{-}1}{2} x = 7$ ____________________

(e) $\frac{x}{3} = 79$ ____________________

2. (Section 17-2.)
For each equation, write what to do to both sides in order to get x by itself on the left side.

(a) $3x + 9 = {}^{-}21$ First ____________________

and then ____________________

(b) $\frac{1}{4} x + {}^{-}15 = 60$ First ____________________

and then ____________________

(c) $\frac{5x}{16} + 49 = 752$ First ____________________

and then ____________________

(d) $2x - 3 = 10$ First ____________________

and then ____________________

and then ____________________

3. (Section 17-3.)

Fill the blanks.

(a) In the equation $\frac{3}{x} + 4 = 2$ you know that x cannot be ________.

(b) $\frac{4}{x}$ is the same as $4 \cdot$ ________.

(c) If $\frac{1}{x+3} = \frac{1}{4}$, then $x + 3 =$ ________.

(d) If $\frac{2}{3x} = \frac{1}{6}$, then $3x =$ ________. (Remember the comparison property.)

4. (Section 17-4.)

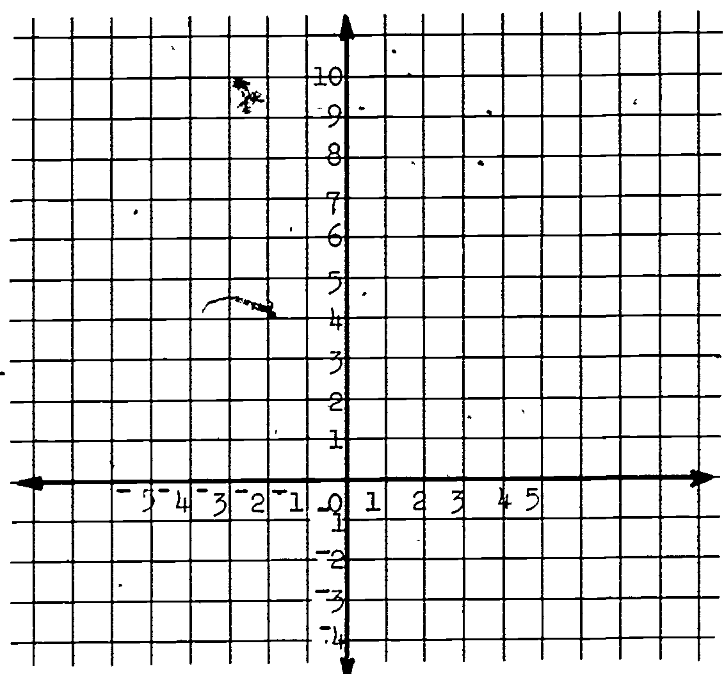
Solve the equation $2x + {}^{-}1 = x + 3$ by writing two functions, graphing them and finding the solution. Check your solution in the equation.

$f : x \rightarrow$ ____________

$g : x \rightarrow$ ____________

$x =$ ________

Check: ____________



5. (Section 17-5.)

Write a chain of equivalent equations to solve these equations:

(a) $^{-}4x + 3 = 2x + 21$ Check: ____________

____

____

____

$x =$ ______

(b) $7x - 4 = 2x + 21$ Check: ____________

____

____

____

____

$x =$ ______

6. (Section 17-6.)

Give the solution set of each inequality.

(a) $2x + 5 < 17$ ____________

(b) $7x + 3 < 2$ ____________

(c) $\frac{x}{5} \leq 7$ ____________

(d) $4x + 1 > 22$ ____________

7. (Section 17-7.)

Solve each inequality by graphing two functions. Show the solution set on the number line below the graphs.

(a) $5x + 2 < 12$

____________ (Equation)

f : x → ____________

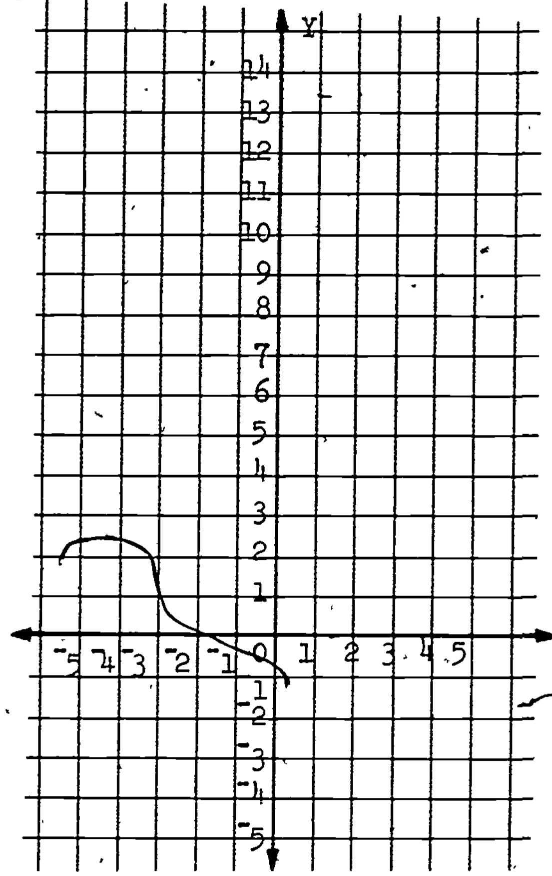
g : x → ____________

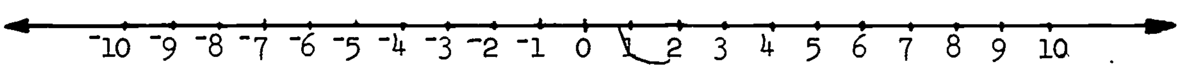
Solution of equation:

x = ________

Write the solution set of the inequality using symbols.

x ________________





(b) $2x + {}^{-}3 \geq 1$

____________ (Equation)

f : x → ____________

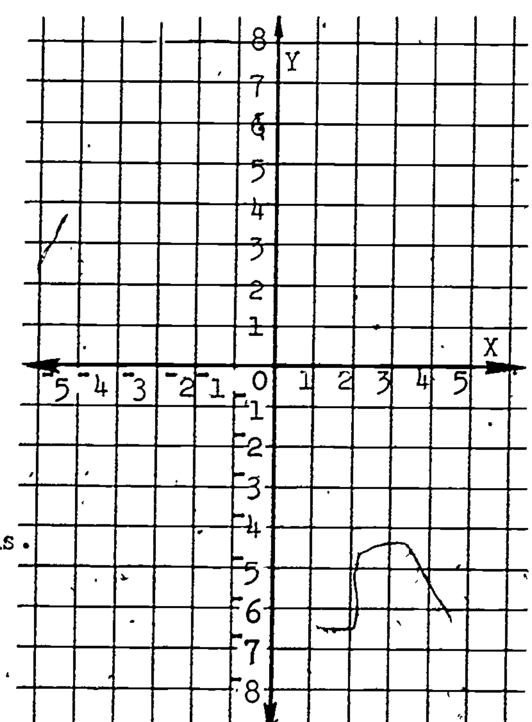
g : x → ____________

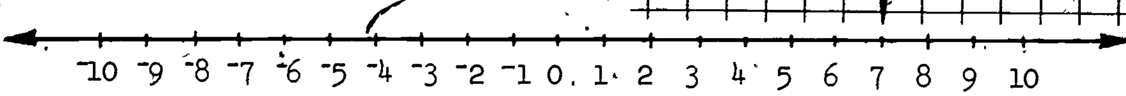
Solution of equation:

x = ________

Write the solution set of the inequality using symbols.

x ________________





(c) $3x + {}^{-}2 < 2x + 1$

______________ (Equation)

$f : x \rightarrow$ ______________

$g : x \rightarrow$ ______________

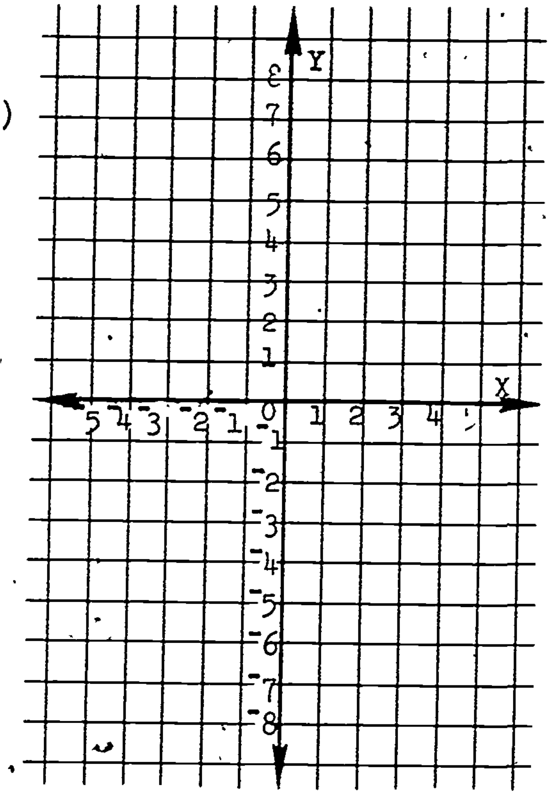
Solution of equation:

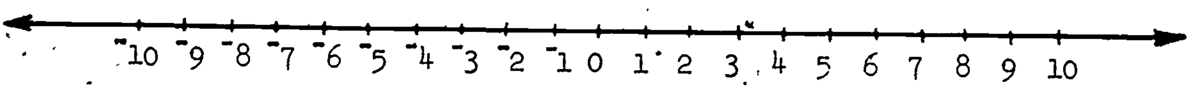
$x =$ ________

Solution set of

$3x + {}^{-}2 < 2x + 1$:

x ________________________





8. (Section 17-8.)

Write equivalent inequalities to solve these. Check using a number from your solution set.

(a) $4x + 3 < x + 6$ Check: ______________

____

____

____

x ______________

(b) $3x + 8 < 5x - 2$ Check: ______________

____

____

____

x ______________

9. Solve this equation. The graph of $f : x \rightarrow 2x^2$ is drawn for you.

$2x^2 - x - 3 = 0$

________________ (Addition equation)

$2x^2 =$

$f : x \rightarrow$ __________

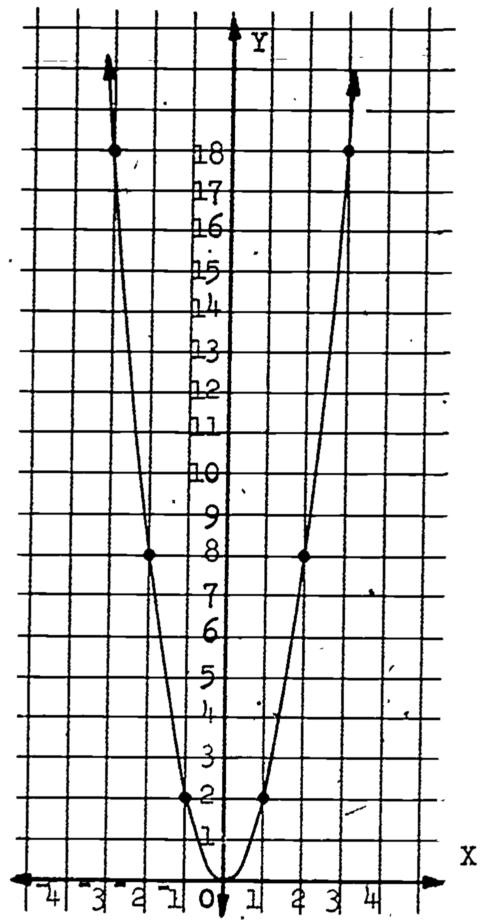
$g : x \rightarrow$ __________

$x =$ ______ or $x =$ ______

(One solution is not an integer.)

Check: ________________

and: ________________



$f : x \rightarrow 2x^2$

Test

1. For each equation, write what to do to both sides of the equation to get x by itself on the left side of the equal sign.

   (a) $x + 16 = 25$ ____________________

   (b) $3x = 17$ ____________________

   (c) $\frac{x}{4} = 21$ ____________________

   (d) $x + {}^{-}3 = 4$ ____________________

   (e) $\frac{1}{5}x = \frac{3}{2}$ ____________________

2. Solve these equations by writing equivalent equations. Check the solution in the equation.

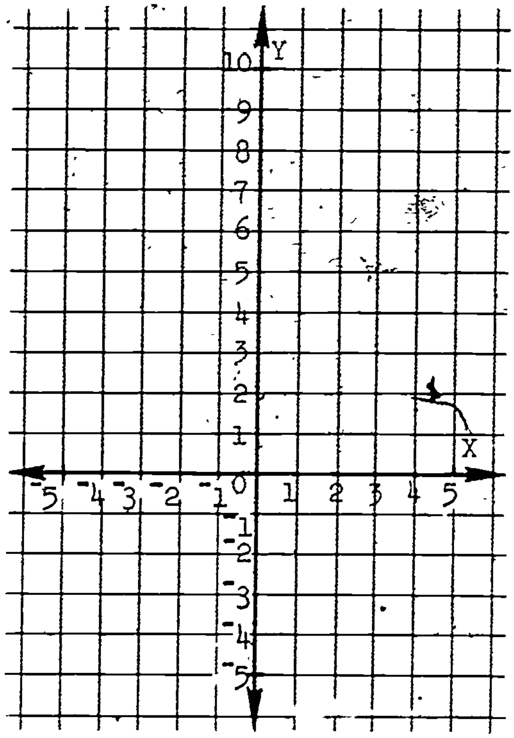
   Check

   (a) $13x + 4 = 30$

   (b) $\frac{5}{3}x - 6 = 19$

   (c) $\frac{3}{2x} + 8 = \frac{83}{10}$

3. Solve these equations by graphing two functions. Check your solution in the equation.



(a) $4x + {}^{-}1 = 7$

$f : x \rightarrow$ ______________

$g : x \rightarrow$ ______________

$x =$ ______

Check:

____

(b) ${}^{-}3x - 2 = 2x + {}^{-}7$

____

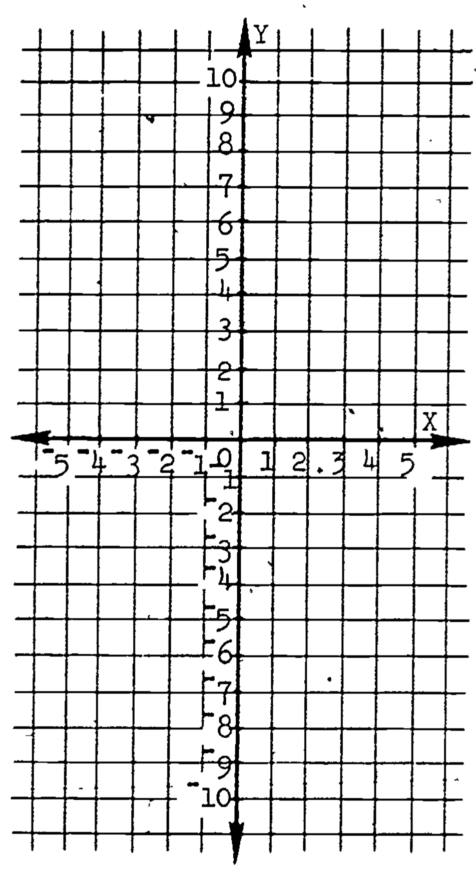
$f : x \rightarrow$ ______________

$g : x \rightarrow$ ______________

$x =$ ______

Check:

____



4. Solve this equation by writing equivalent equations. Check your solution in the equation.

Check:

$3x - 2 = {}^{-}x + 10$

$x =$ ________

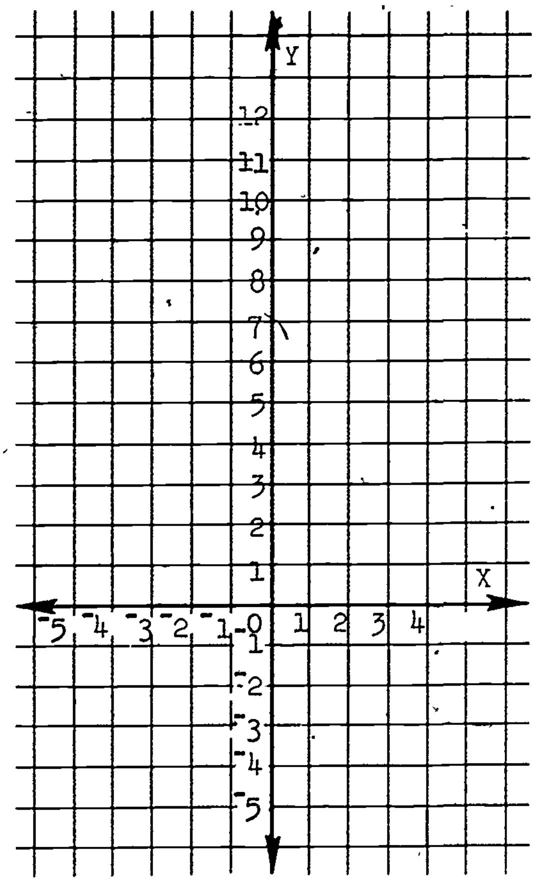
5. Solve this inequality by graphing two functions. Show the solution set on the number line.

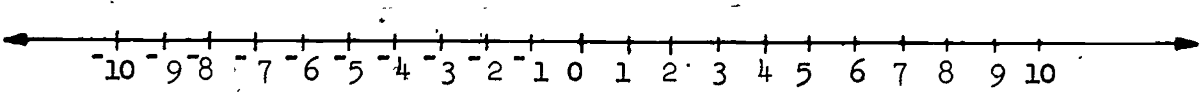
$x + {}^{-}3 > 3x + 1$

________________ (Equation)

$f : x \rightarrow$ __________

$g : x \rightarrow$ __________





6. Solve this inequality by writing equivalent inequalities. Check a number of your solution set in the inequality.

$^{-}x + 2 \geq x + 3$ Check: ______________

____

____

____

x______________

7. Solve the equation $x^2 - 2x - 3 = 0$. The function $x \to x^2$ is shown.

$x^2 - 2x - 3 = 0$

____

____

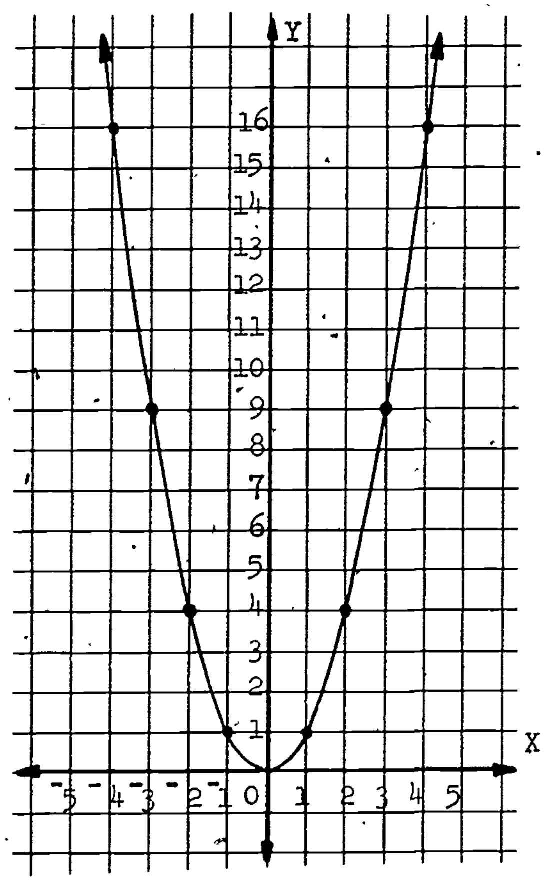
$f : x \to$ ______________

$g : x \to$ ______________

$x =$ ______ or $x =$ ______

Check: ______________

and: ______________



$f : x \to x^2$

Check Your Memory: Self-Test

1. (Section 13-6.)

   Add. (You may use your flow chart if you need it.)

   (a) $\frac{3}{4} + \frac{1}{2} =$ ______

   (b) $\frac{9}{8} + \frac{^{-}1}{4} =$ ______

   (c) $\frac{4}{5} + \frac{2}{3} =$ ______

   (d) $\frac{5}{6} + \frac{3}{4} =$ ______

   (e) $\frac{2}{5} + \frac{^{-}1}{4} =$ ______

2. (Section 13-8.)

   Use exponents to rewrite each problem. Give the answer with an exponent.

   (a) $1000 \times .0001 =$ ______ $\times$ ______

   $=$ ______

   (b) $\dfrac{100000}{\frac{1}{100}} =$ ______

   $=$ ______

   (c) $.00001 \times .001 =$ ______ $\times$ ______

   $=$ ______

3. (Section 13-10.)

Write each number using scientific notation. (Use your flow chart if you need it.)

(a) 346795 = ____________

(b) 228.167 = ____________

(c) .00041 = ____________

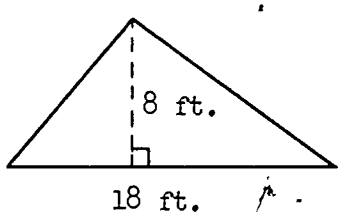
(d) 455000 = ____________

(e) .0036872 = ____________

4. (Section 15-13.)

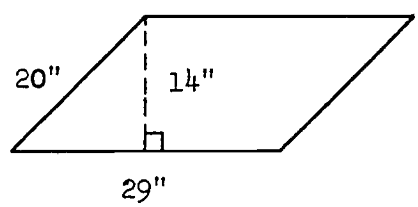
Find the area of these figures.

(a)



AREA ____________

(b)



AREA ____________

5. (Sections 15-7 and 15-8.)

(a) How many inches are in $2\frac{1}{2}$ feet? ____________

(b) How many yards are in 2 feet? ____________

(c) How many inches are in $3\frac{1}{4}$ yards? ____________

6. (Sections 16-7 and 16-8.)

(a) Find the circumference of a circle with a radius of 10 feet.
(Use $\pi \approx 3.14$.) ____________

(b) Find the area of the circle in (a), above. ____________

Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1. (a) $\frac{5}{4}$

   (b) $\frac{7}{8}$

   (c) $\frac{22}{15}$

   (d) $\frac{19}{12}$

   (e) $\frac{3}{20}$

2. (a) $10^3 \times 10^{-4} = 10^{-1}$

   (b) $\frac{10^5}{10^{-2}} = 10^7$

   (c) $10^{-5} \times 10^{-3} = 10^{-8}$

3. (a) 3.46795×10^5

   (b) 2.28167×10^2

   (c) 4.1×10^{-4}

   (d) 4.55×10^5

   (e) 3.6872×10^{-3}

4. (a) 72 sq. ft.

   (b) 406 sq. in.

5. (a) 30 inches

   (b) $\frac{2}{3}$ yd.

   (c) 117 inches

6. (a) 62.8 feet

   (b) 314 sq. ft.

Chapter 18

COORDINATE GEOMETRY

Chapter 18

COORDINATE GEOMETRY

Slope of a Line

You have seen the graphs of the functions

$$k : x \to \frac{1}{2}x$$

$$i : x \to 1x$$

$$f : x \to 2x$$

$$g : x \to 3x$$

$$h : x \to 4x$$

and so on

many times before. You know that the graphs of these functions

(1) are lines,

and (2) pass through the origin.

On the coordinate plane on the next page are the graphs of the functions:

$$k : x \to \frac{1}{2}x$$

$$i : x \to 1x$$

$$f : x \to 2x$$

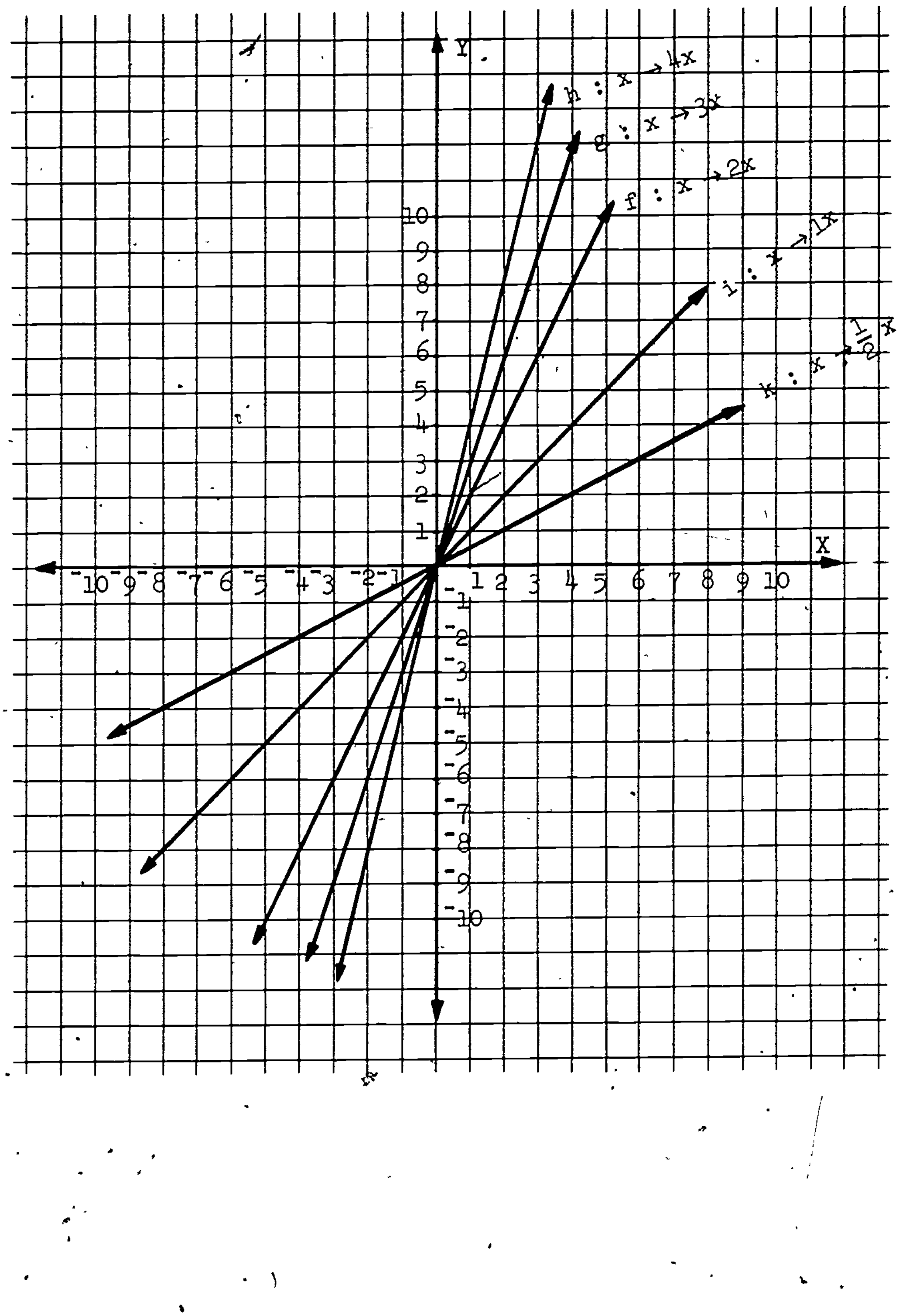
$$g : x \to 3x$$

$$h : x \to 4x$$

Class Discussion

Imagine that an ant starts at the origin and crawls "up" the line of one of the functions.

Look at all five of the lines. Which line is steepest for the ant to crawl up? __________ Which line is least steep? __________


Y
X
h : x → 4x
g : x → 3x
f : x → 2x
i : x → 1x
k : x → $\frac{1}{2}$x

For these kinds of functions, as the number that multiplies x gets larger the line gets ________________.
(steeper, less steep)

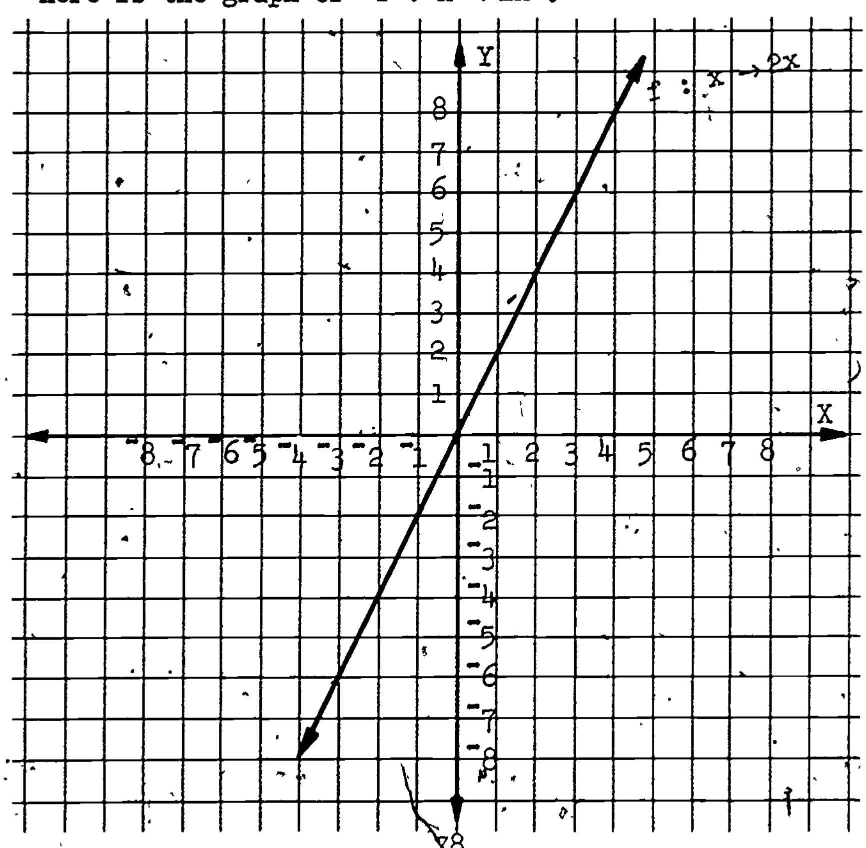
Which line is steeper?

1. $g : x \to 3x$ or $i : x \to 1x$? ____________
2. $f : x \to 2x$ or $k : x \to \frac{1}{2}x$? ____________
3. $j : x \to 10x$ or $p : x \to 14x$? ____________
4. $q : x \to \frac{1}{4}x$ or $r : x \to \frac{3}{4}x$? ____________
5. $t : x \to \frac{2}{3}x$ or $s : x \to \frac{3}{2}x$? ____________

The measure of the steepness of a line is what we call the <u>slope</u> of a line. The number that multiplies x is the <u>slope</u> of the line.

Notice in all the examples so far the number that multiplied x was <u>positive</u>. Also notice that as you move "up" on the graph the line goes farther to the right. The slope is <u>positive</u> for these lines.

Here is the graph of $f : x \to 2x$.



The slope of $f : x \rightarrow 2x$ is 2.

Is the origin $(0,0)$ on the line $f : x \rightarrow 2x$? ________

Starting at the origin, count up 2 units on the Y-axis and then count over to the right 1 unit. Is this point on the line?

____

When we start on a line and move up and then over to the line again, we call the distance "up" the <u>rise</u> and the distance over to the line the <u>run</u>. If we count over to the <u>right</u> to get back to the line the <u>run</u> is <u>positive</u>. If we count over to the <u>left</u> to get back to the line the <u>run</u> is <u>negative</u>.

Starting at the point $(1,2)$, count up 2 units. How many units do you count over to the right to come back to a point on the line? ________ What is the rise? ________ What is the run?

____

Starting at the point $(^-4, ^-8)$, count up 2 units. How many units do you count over to the right to come back to a point on the line? ________ What is the rise? ________

If you start at <u>any</u> point on the line, count up 2 units and then count over to the right one unit, do you come back to the line? ________ (Try this and see.)

For the line $f : x \rightarrow 2x$, in each case the rise is 2 and the run is 1.

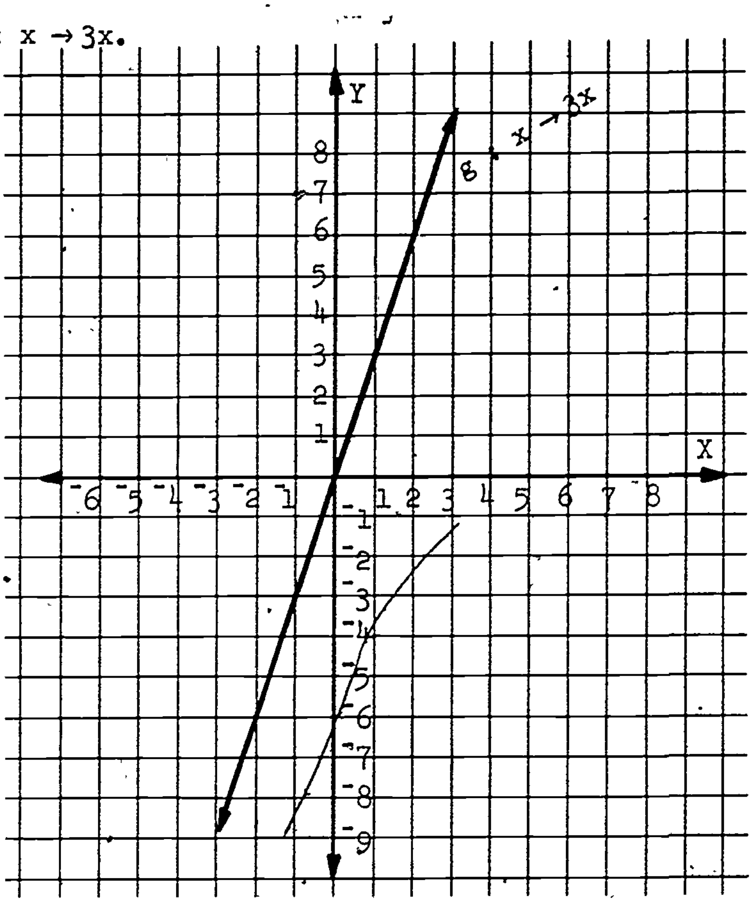
Suppose we divide the rise by the run. $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$ This is the slope of the line.

Look at the graph of $f : x \rightarrow 2x$ again. Start at the origin and count up 4 units. How many units do you count over to the right to come back to the line? ________ The rise is ________ and the run is ________. Now divide the rise by the run.

$\frac{\text{rise}}{\text{run}} = \text{———} = $ ________ Is this answer the slope? ________

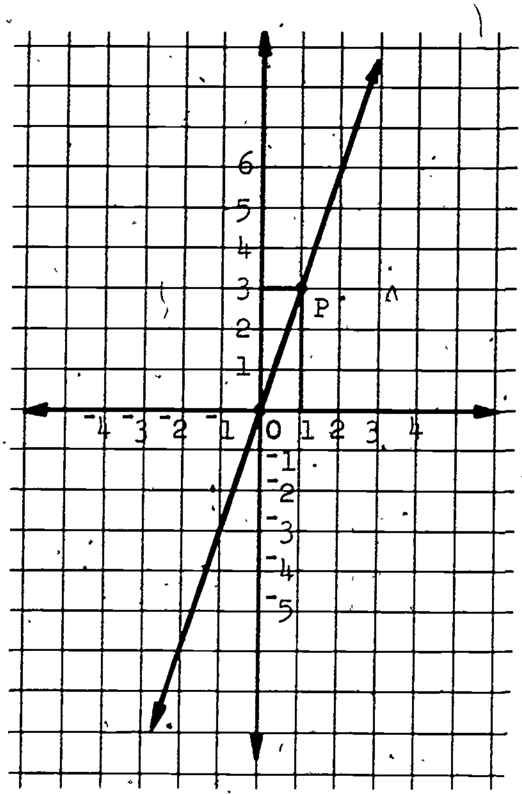
So far the rise divided by the run has given us the slope of the line. Let's see if this works on another line.

Here is the graph of $g : x \rightarrow 3x$.



What is the slope of the line $g : x \rightarrow 3x$?

________ It will help you find the rise and run if you think of different segments of this line as being the diagonals of different rectangles.

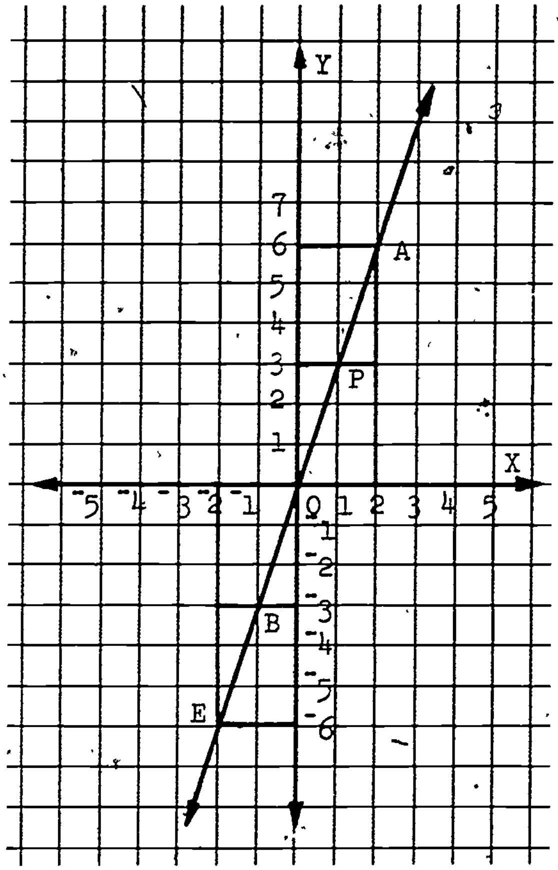


On the graph label the origin O and label the point (1,3) P. $\overline{OP}$ is the diagonal of a rectangle.

By looking at the rectangle it is easy to see that the rise is 3 and the run is 1, and $\frac{\text{rise}}{\text{run}}$ = ——— = ________ . Does this give you the slope?

____

Here is the line $g : x \to 3x$ with some more rectangles drawn in.



Look at the rectangle with diagonal $\overline{OA}$. What is the rise? ________ What is the run? ________ $\frac{\text{rise}}{\text{run}} = \underline{\quad} = $ ________. Does this give you the slope? ________.

Look at the rectangle with diagonal $\overline{PA}$. $\frac{\text{rise}}{\text{run}} = \underline{\quad} = $ ________.

Look at the rectangle with diagonal $\overline{BO}$. $\frac{\text{rise}}{\text{run}} = \underline{\quad} = $ ________.

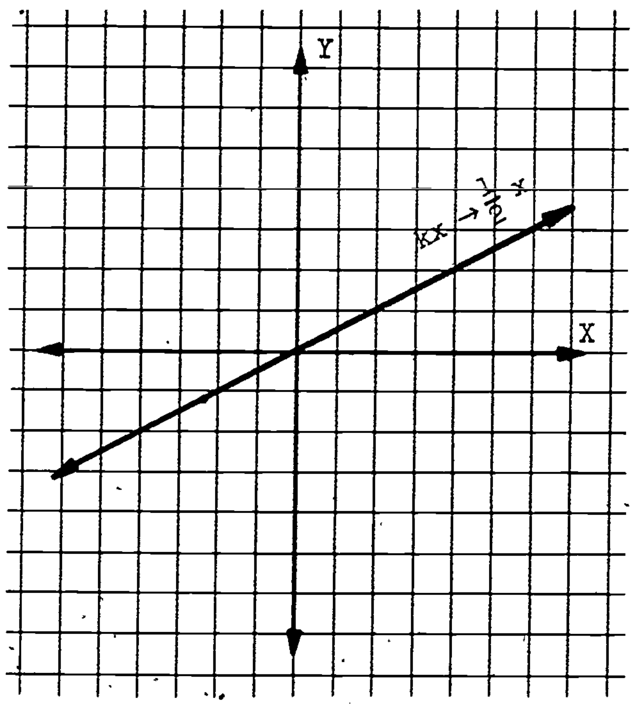
Look at the rectangle with diagonal $\overline{EO}$. $\frac{\text{rise}}{\text{run}} = \underline{\quad} = $ ________.

Look at the rectangle with diagonal $\overline{EB}$. $\frac{\text{rise}}{\text{run}} = \underline{\quad} = $ ________.

In each of these cases $\frac{\text{rise}}{\text{run}} = 3$ which is the slope.

Does it now seem more likely that $\frac{\text{rise}}{\text{run}}$ always gives the slope of the line? ________

Try this for the line $k : x \to \frac{1}{2} x$.

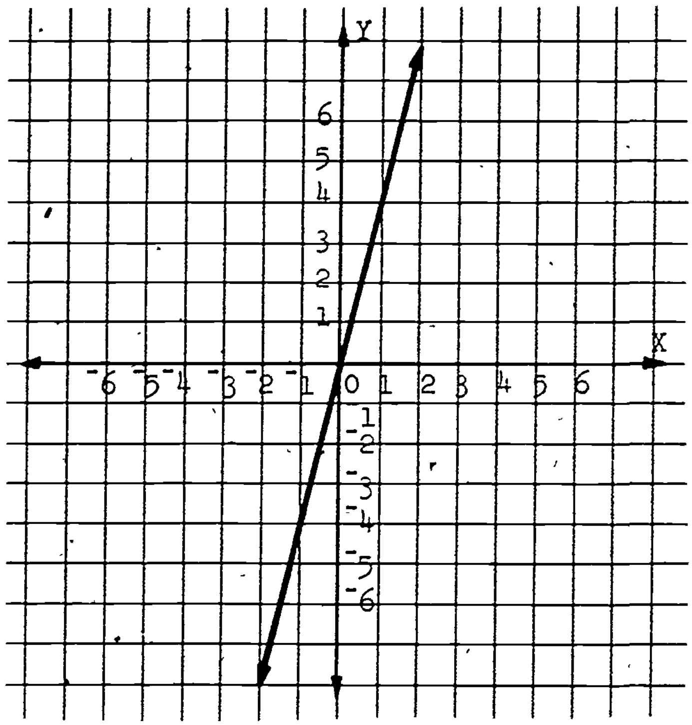


You can pick any two points on the line for endpoints of the diagonal but it is easier to find the rise and run if you pick points that have integer coordinates.

Pick two points on the line that have integer coordinates. Use the segment between your points as the diagonal of a rectangle. Draw this rectangle on the graph of $k : x \to \frac{1}{2} x$. What is the rise? ________ What is the run? ________ $\frac{\text{rise}}{\text{run}} =$ ____ . Do you get $\frac{1}{2}$? ________ Is $\frac{1}{2}$ the slope of this line? ________

Exercises

1.

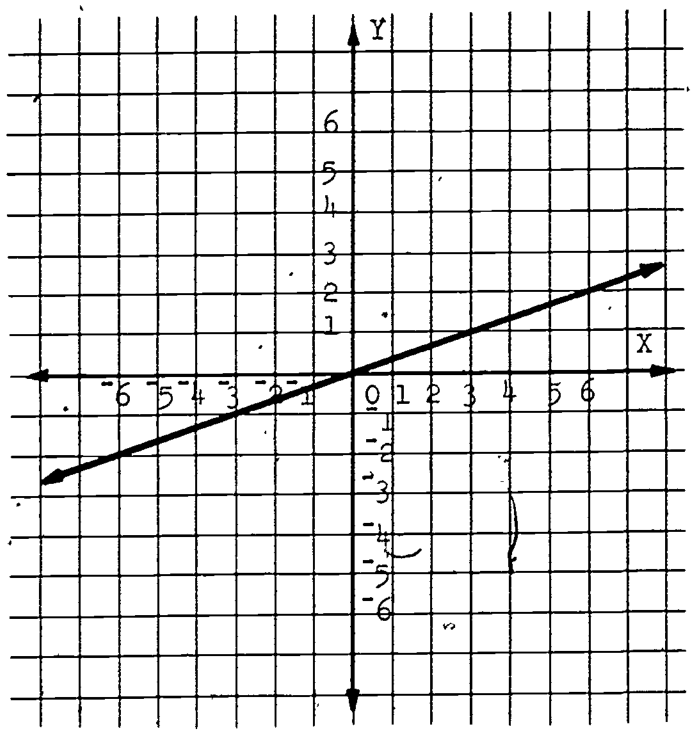


(a) What is the rise of the line? ________

(b) What is the run of the line? ________

(c) What is the slope of the line? ________

2.



(a) What is the rise of the line? ________

(b) What is the run of the line? ________

(c) What is the slope of the line? ________

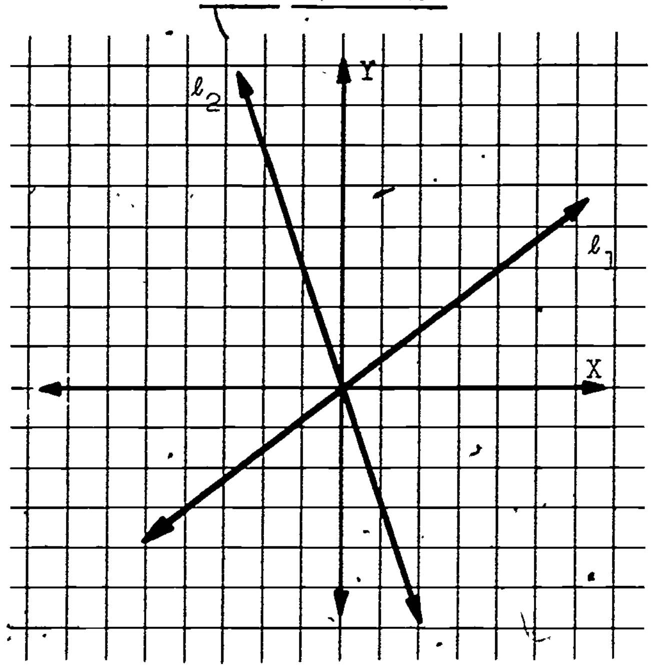
Lines with Negative Slope

In the last lesson all the lines leaned to your right, and they all had positive slope.

Lines that lean to your left have negative slope. For these lines you count up to find the rise of the line, but you must count over to the left to find its run.

When you count to the right the run is positive.
When you count to the left the run is negative.

Class Discussion



1. (a) Does ℓ_1 lean to your right or your left? ________

   (b) Is the slope of ℓ_1 positive or negative? ________

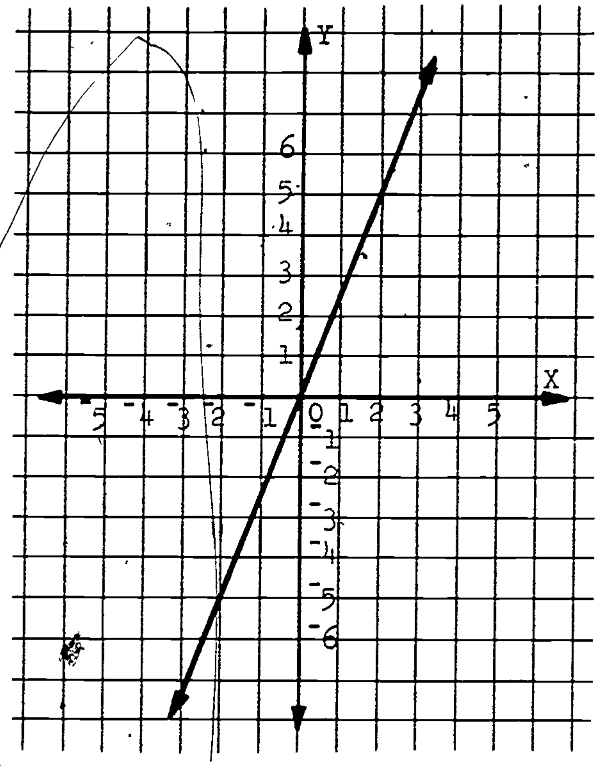
   (c) For ℓ_1 $\frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} =$ ________.

   (d) The slope of $\ell_2 =$ ________.

Exercises

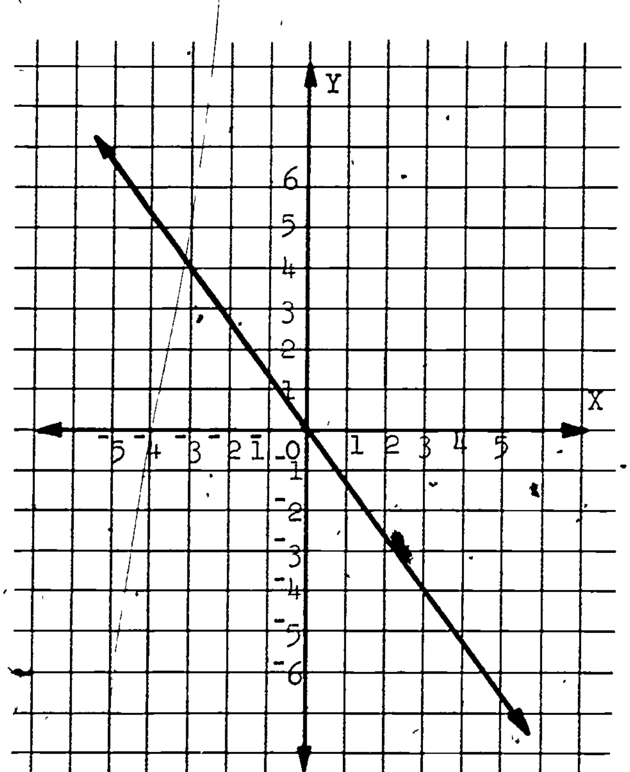
Write the slope of each line by finding its $\frac{\text{rise}}{\text{run}}$.

1.



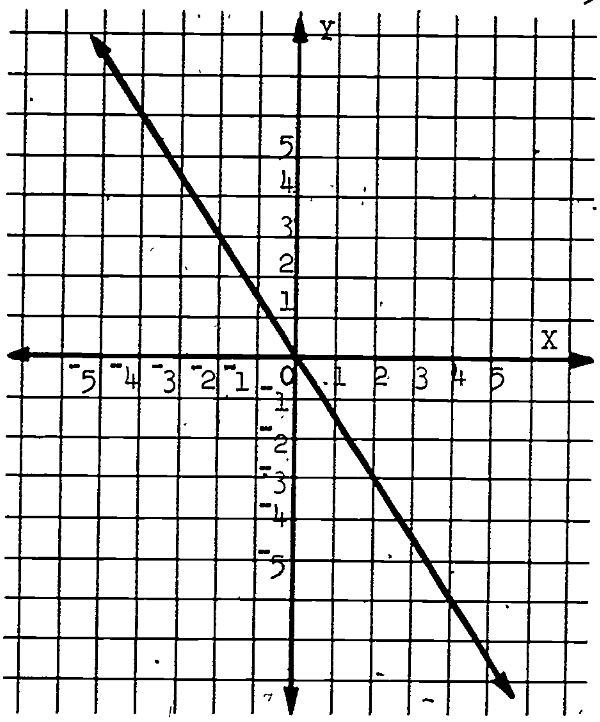
Slope: ______

2.



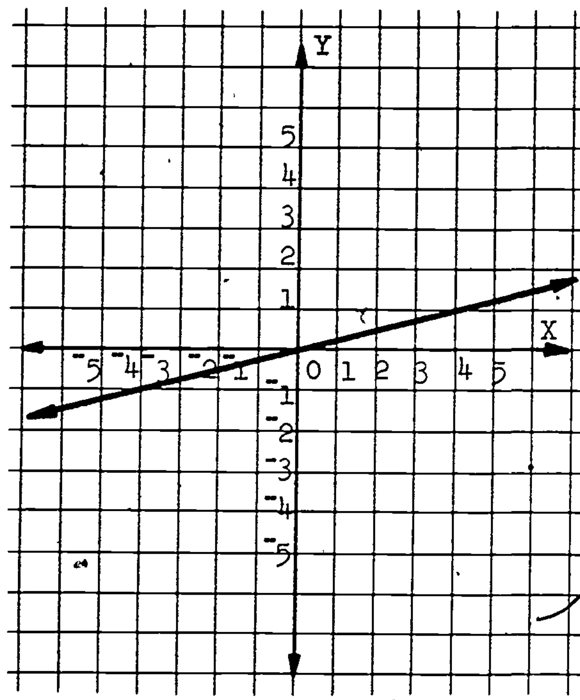
Slope: ______

3.



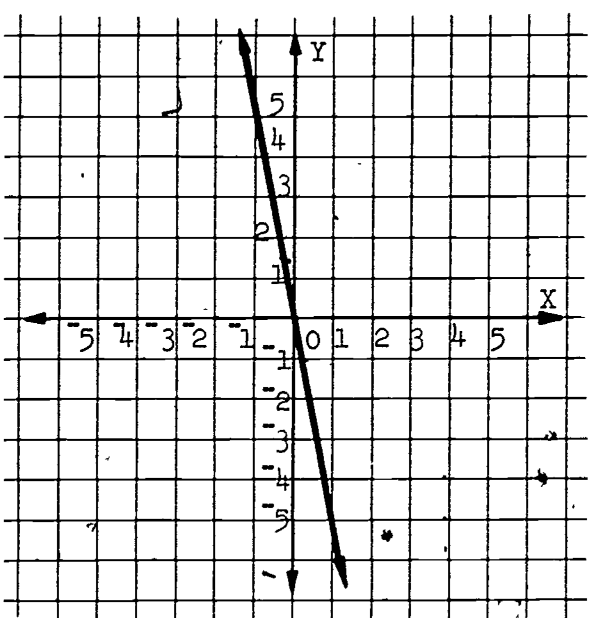
Slope: ________

5.



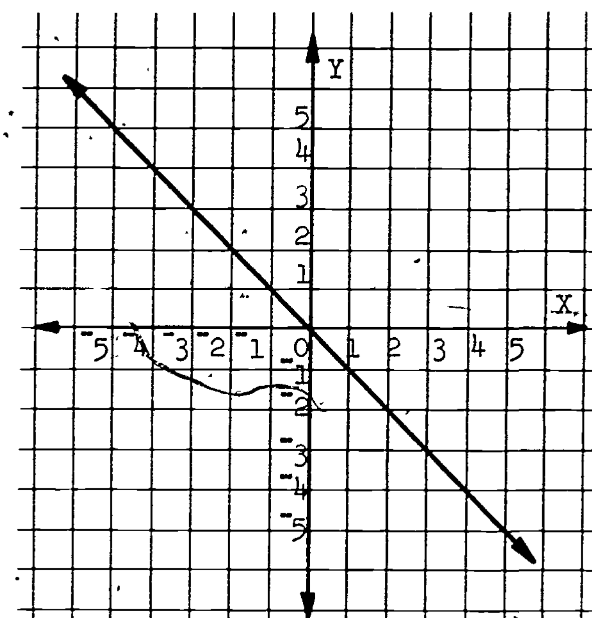
Slope: ________

4.



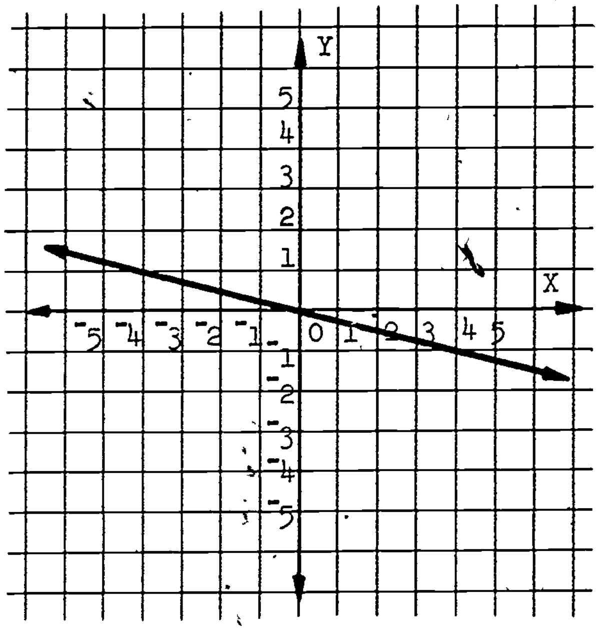
Slope: ________

6.



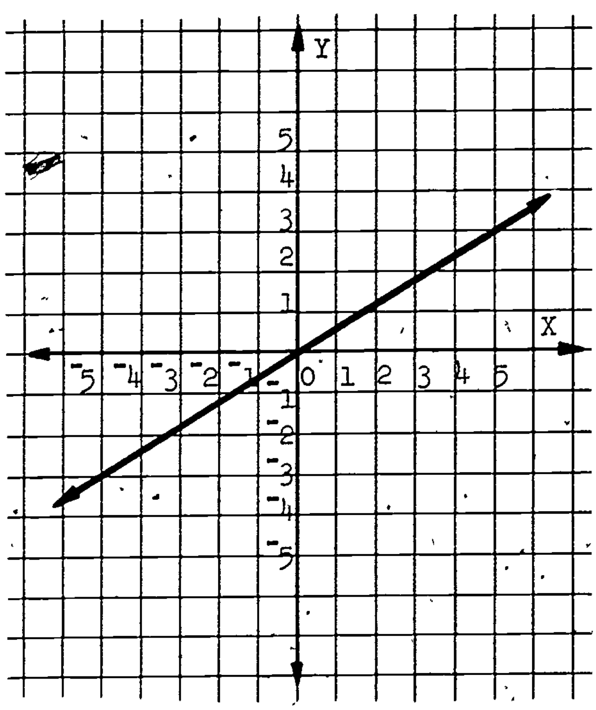
Slope: ________

7.



Slope: ______

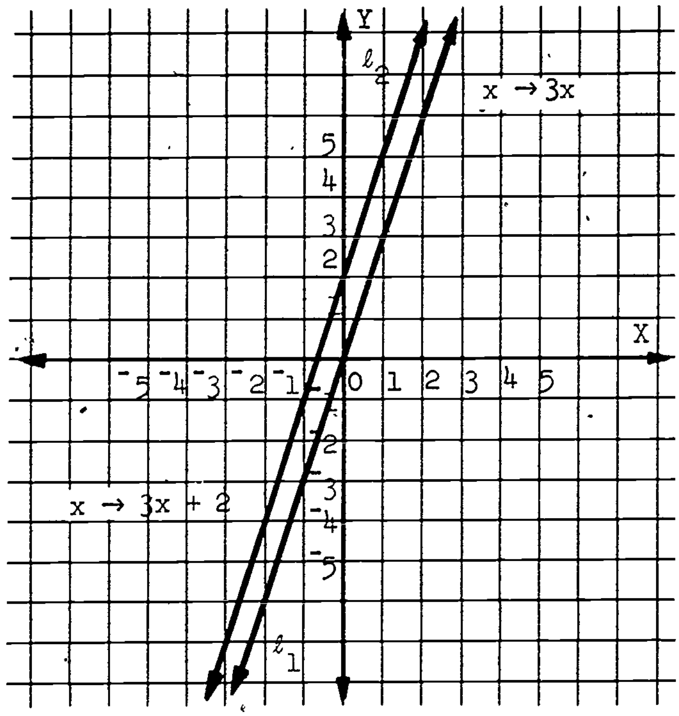
8.



Slope: ______

The Y-Intercept of a Line

The slope of a line that does not pass through the origin is found in the same way as the slope of lines that do pass through the origin.



Here are the graphs of ℓ_1 and ℓ_2. ℓ_1 is the line $f : x \rightarrow 3x$. It has a slope of 3.

What is the rise for ℓ_2 ? __________

What is the run for ℓ_2 ? __________

What is the slope of ℓ_2 ? __________

Even though both ℓ_1 and ℓ_2 have the same slope they are different lines.

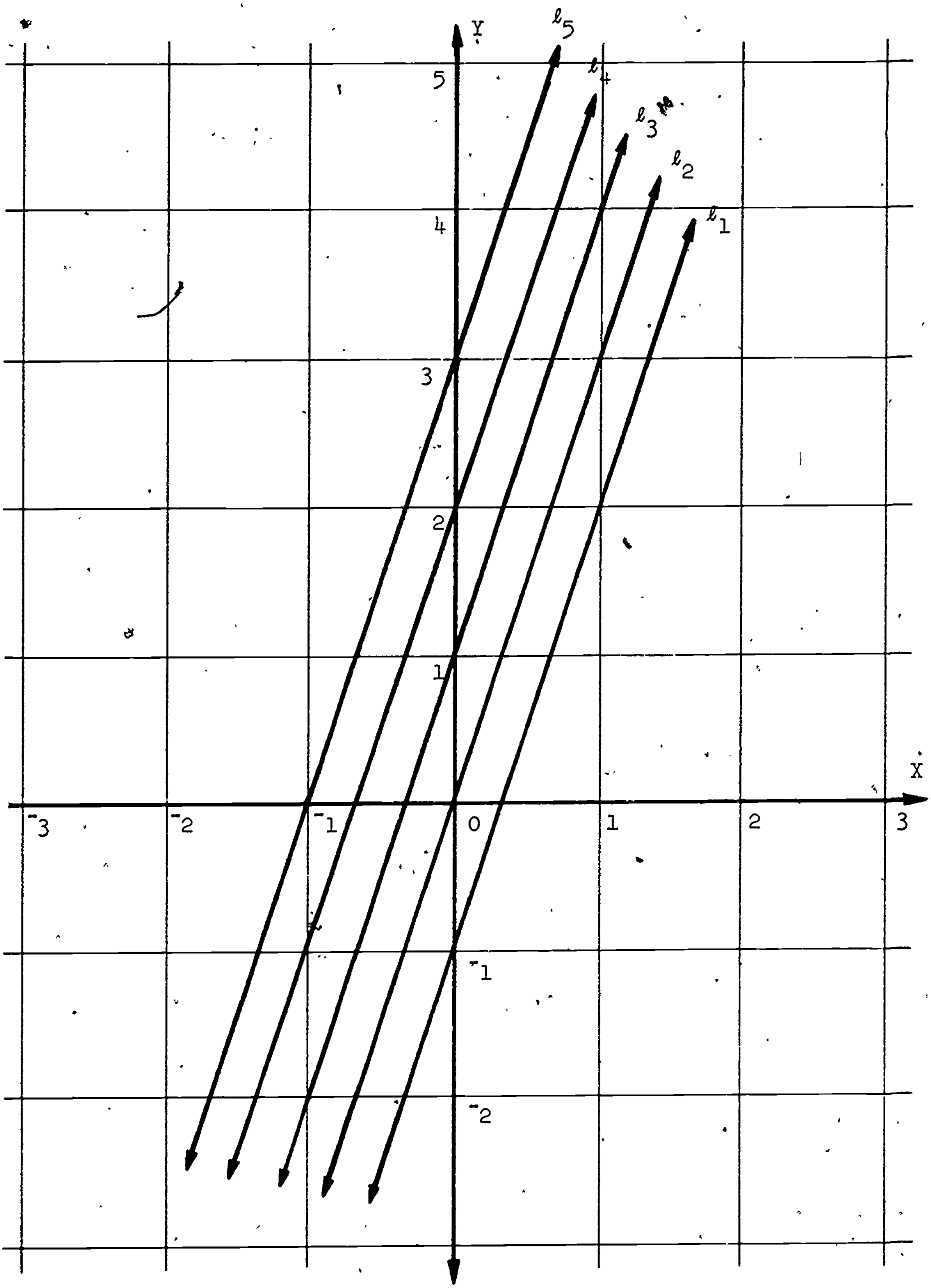
One thing that is different about them is the point where they cross the Y-axis.

What are the coordinates of the point where ℓ_2 crosses the y-axis? (,) What is the y coordinate of that point? __________

> The y coordinate of the point where a line crosses the Y-axis is called the y-intercept.

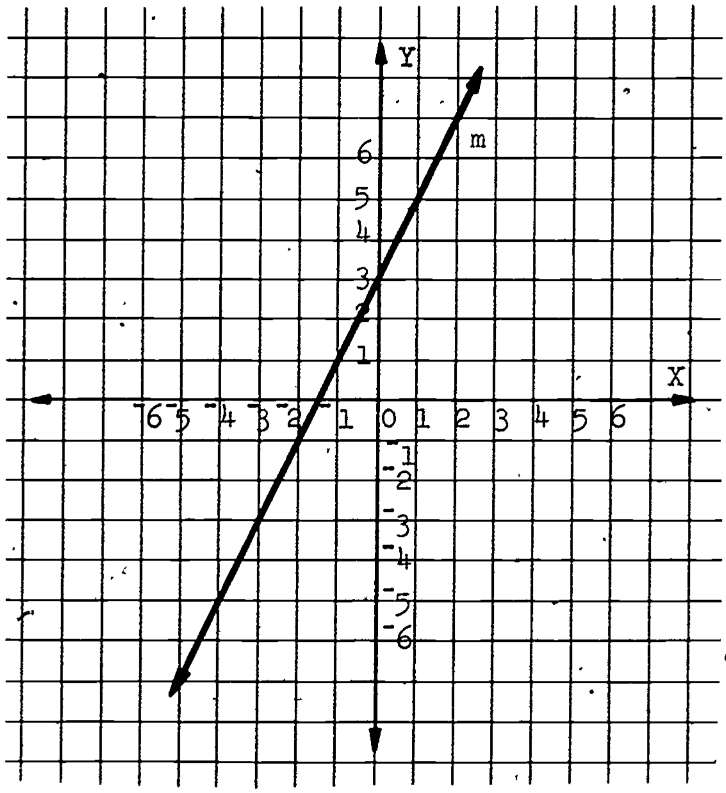
What is the y-intercept of ℓ_1 ? __________

Here are the graphs of ℓ_1, ℓ_2, ℓ_3, ℓ_4, and ℓ_5. They all have a slope of 3.



Class Discussion

1. What is the y-intercept of ℓ_1 ? ________
2. What is the y-intercept of ℓ_2 ? ________
3. What is the y-intercept of ℓ_3 ? ________
4. What ~~is the~~ the y-intercept of ℓ_4 ? ________
5. What is the y-intercept of ℓ_5 ? ________
6. Look at the graph of this line "m" .



(a) What is its y-intercept? ________

(b) What is its slope? ________

(c) Can you draw other lines that have a y-intercept of 3 ? ________

(d) Will any of these other lines also have a slope of 2 ? ________

(e) Can you draw other lines that have a slope of 2 ? ________

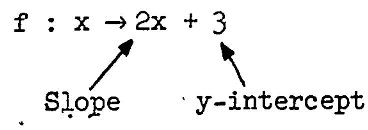
(f) Will any of these other lines also have a y-intercept of 3 ? ________

(g) Is line "m" the only line that has a slope of 2 and a y-intercept of 3 ? ________

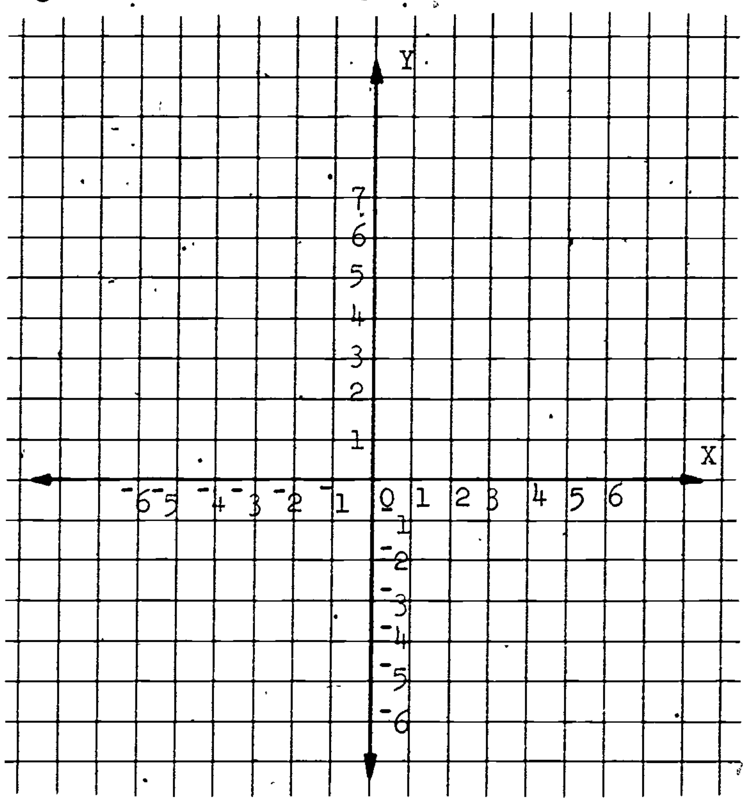
(h). Make a table of three inputs and three outputs for $f : x \to 2x + 3$ and plot the 3 points on the graph of line "m".

(i) Is line "m" the graph of $f : x \to 2x + 3$?

In $f : x \to 2x + 3$ the 2 is the slope and the 3 is the y-intercept.

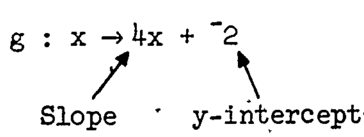


7. (a) Make a table of 3 inputs and 3 outputs for $g : x \to 4x + {}^{-}2$. and graph the function here.



(b) Look at the graph of $g : x \to 4x + {}^{-}2$. What is the slope? __________ What is the y-intercept? __________

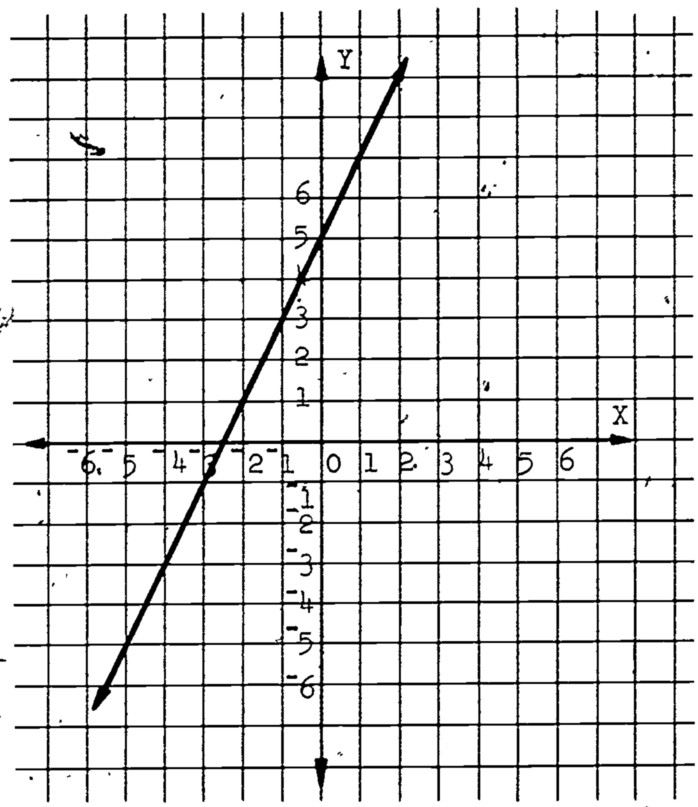
Again you can see from the graph you drew that



Now you can look at the graph of a line, find the slope and the y-intercept, and easily write its function like this:

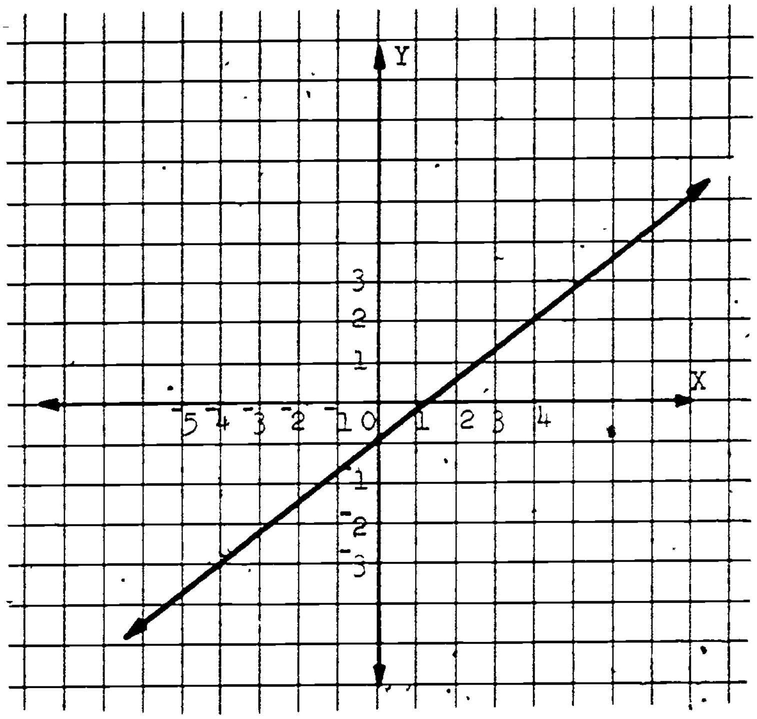
$$f : x \rightarrow (\text{slope})\ x + (\text{y-intercept})$$

8. Write the function for this line.



$f : x \rightarrow$ ____________________

9. Write the function for this line.



f : x → ________________

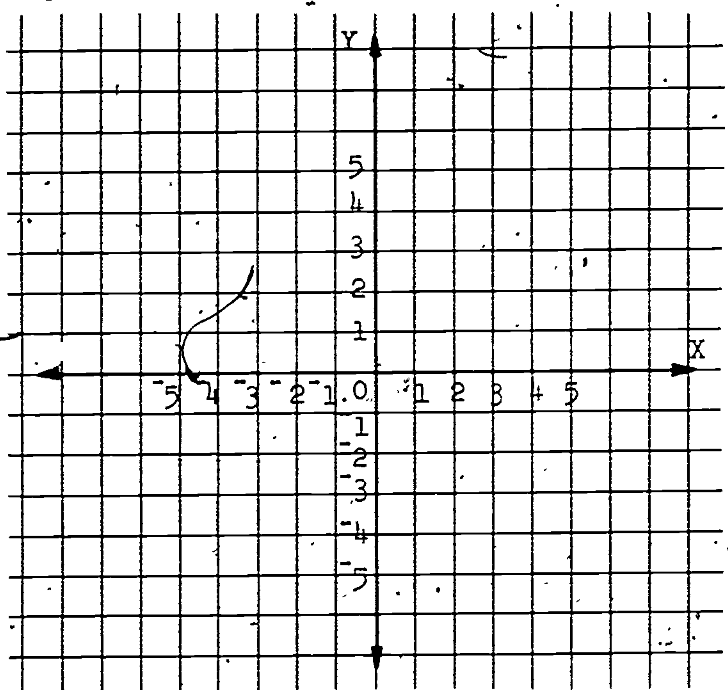
It is easy to look at a function written in arrow form and see what numbers are the slope and y-intercept of the function. Once you know the slope and y-intercept you can draw the graph of the function <u>without making a table of inputs and outputs</u>.

You can plot one point on the line when you know the y-intercept.

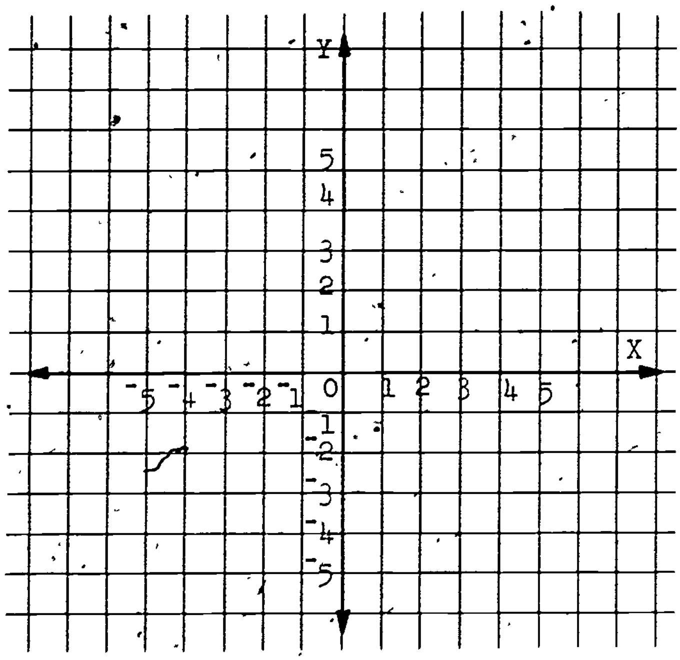
You can start at this point and use the slope $(\frac{\text{rise}}{\text{run}})$ to find other points on the line.

See if you can draw the graphs of these functions without making a table of inputs and outputs.

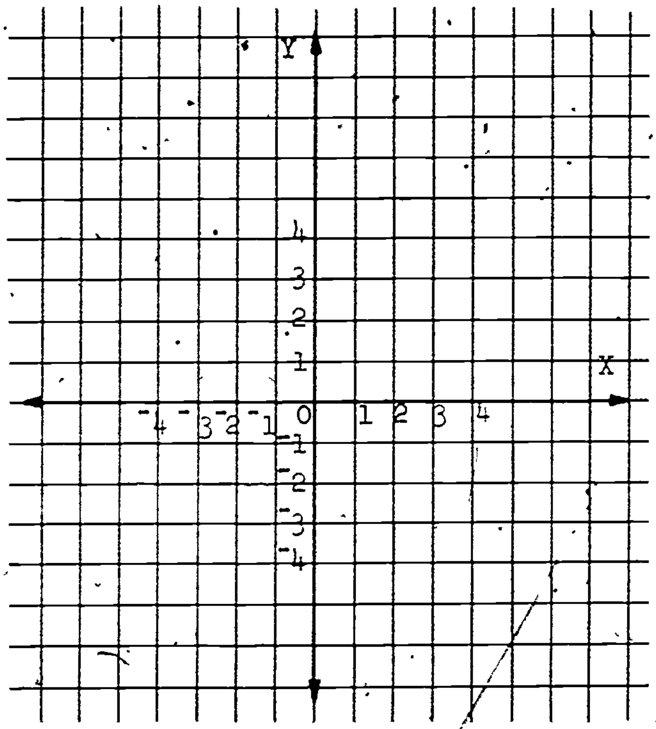
10. $f : x \rightarrow \frac{2}{3}x - 1$ (<u>Hint.</u> Rewrite as addition.)



11. $f : x \rightarrow {}^{-}2x + 3$



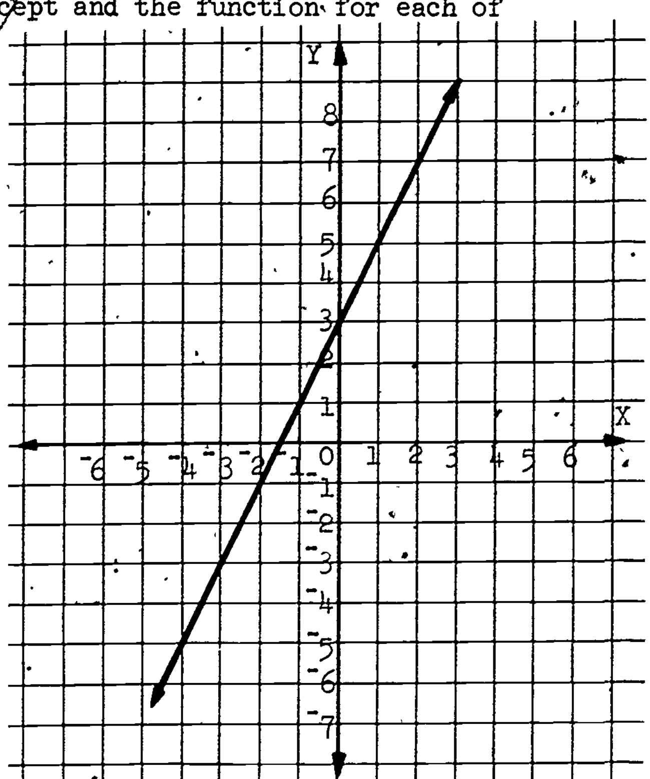
12. $f : x \rightarrow 4x - 2$



Exercises

1. Write the slope, the y-intercept and the function for each of these lines.

(a)

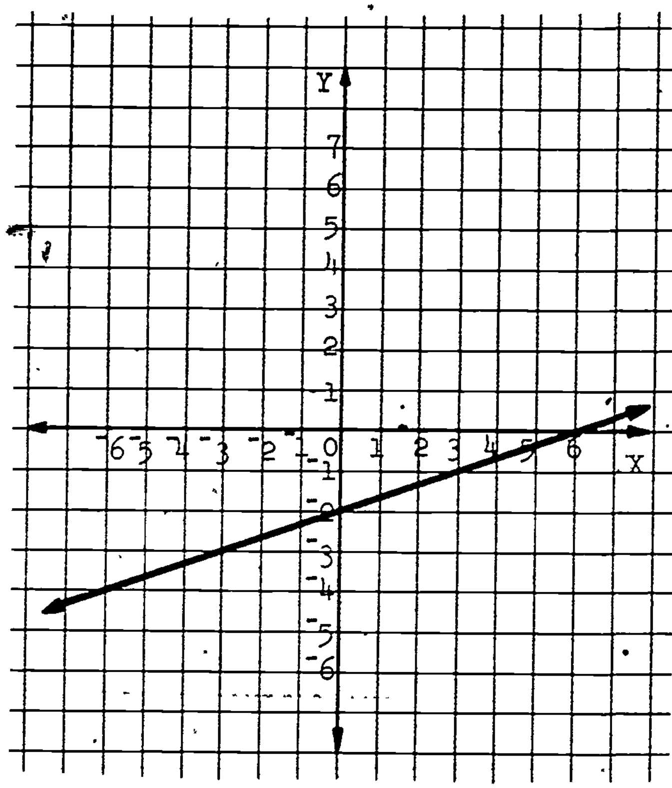


slope = ______

y-intercept is ______

$f : x \rightarrow$ ______

(b)



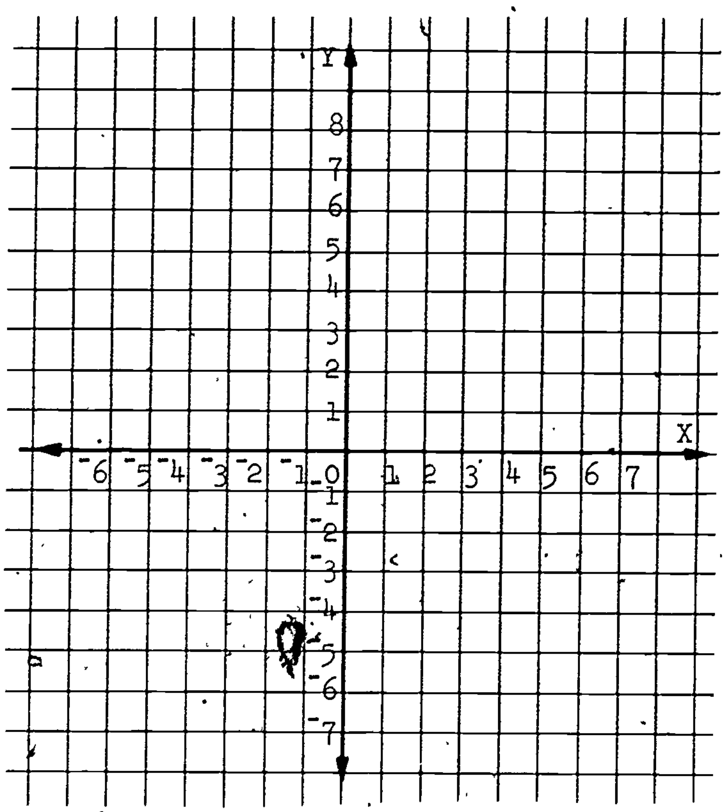
slope = ________

y-intercept is ________

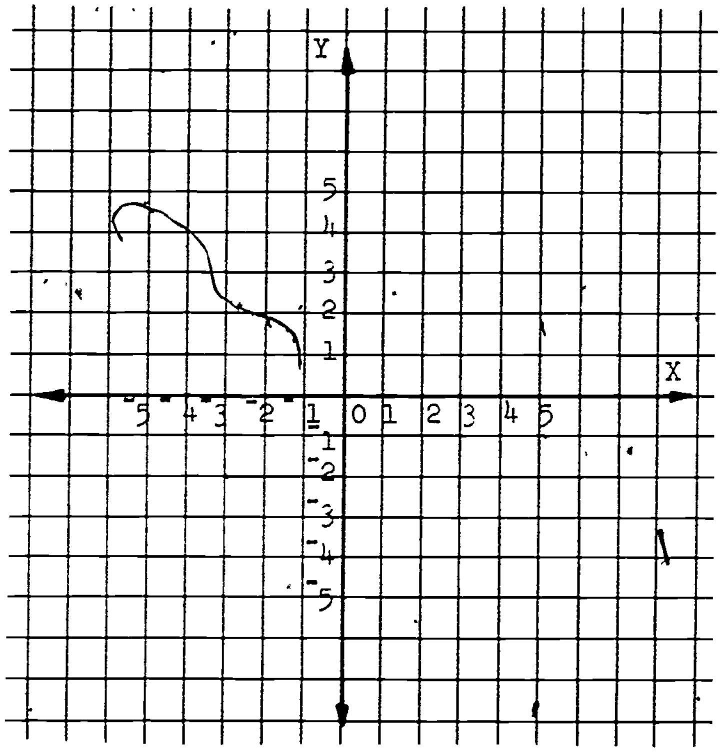
$f : x \rightarrow$ ________________

2. Draw the graph of each of these lines without making a table of inputs and outputs.

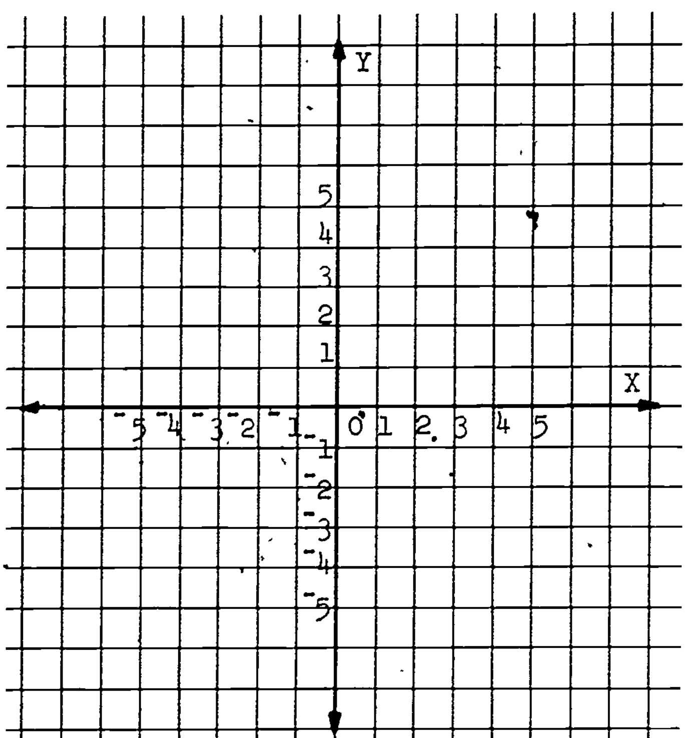
(a) $f : x \rightarrow 5x - 3$



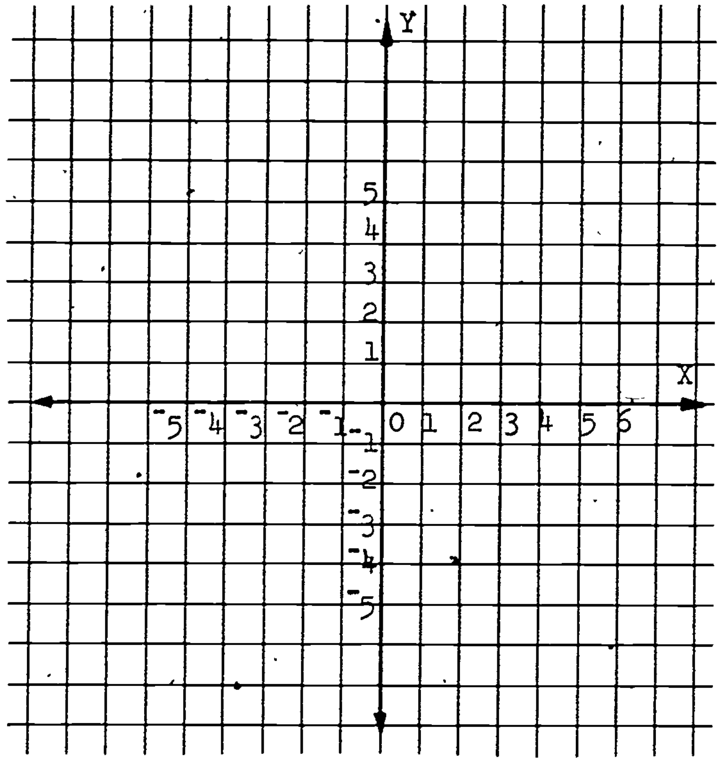
(b) $g : x \rightarrow \frac{1}{4}x + 5$



(c) $h : x \rightarrow \frac{^-3}{4}x + 1$



(d) $k : x \to {}^-2x - 3$



The Equation of a Line

The graph of this function is a line.

$$f : x \rightarrow 3x + 2$$

Class Discussion

1. What is the input of the function? ____________
2. What is the output of the function? ____________
3. Which is the output axis in the coordinate plane? ____________ (X or Y)
4. When you write the coordinates of a point, like (2,8) , or (1,5) , or (x,y) which coordinate of the pair is the output?

____________ (x or y)

You see that:

$$\underbrace{(x,y)}_{\text{output of the pair}} \qquad f : x \rightarrow \underbrace{3x + 2}_{\text{output of the function}}$$

Since $3x + 2$ is the output and y is also the output they are equal. So,

$$y = 3x + 2$$

Here is another example:

$$\underbrace{(x,y)}_{\text{output of the pair}} \qquad g : x \rightarrow \underbrace{\frac{3}{4}x - 5}_{\text{output of the function}}$$

So:

$$y = \frac{3}{4}x - 5$$

Here is a third example:

$$\underbrace{(x,y)}_{\text{output of the pair}} \qquad k : x \to \underbrace{{}^{-}\tfrac{1}{2}x + 3}_{\text{output of the function}}$$

So:

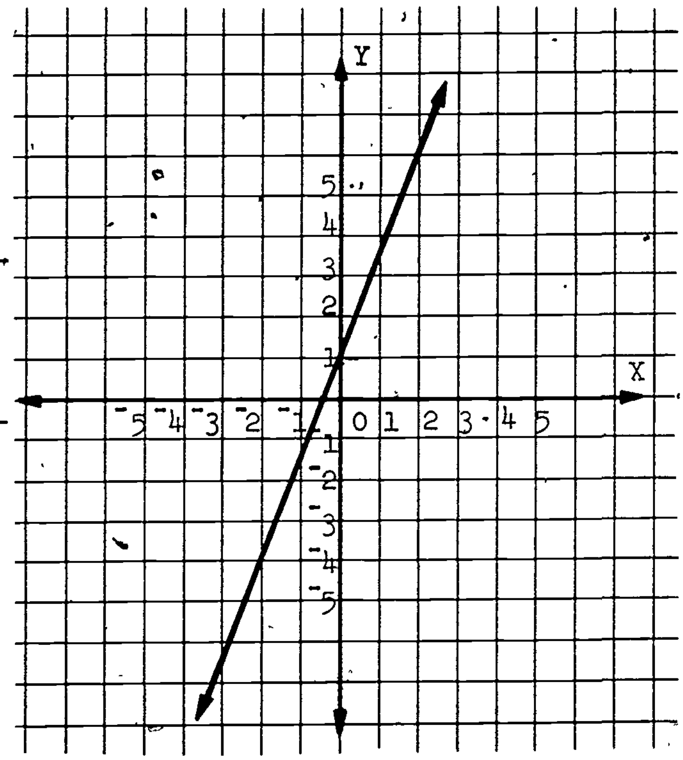
$$y = {}^{-}\frac{1}{2}x + 3$$

function whose graph is a line	equation of the line
$f : x \to 3x + 2$	$y = 3x + 2$
$g : x \to \frac{3}{4}x - 5$	$y = \frac{3}{4}x - 5$
$k : x \to {}^{-}\frac{1}{2}x + 3$	$y = {}^{-}\frac{1}{2}x + 3$

Equations like these whose graphs are lines are called <u>linear</u> equations.

Exercises

1.



(a) What is the slope of this line? __________

(b) What is the y-intercept of this line? __________

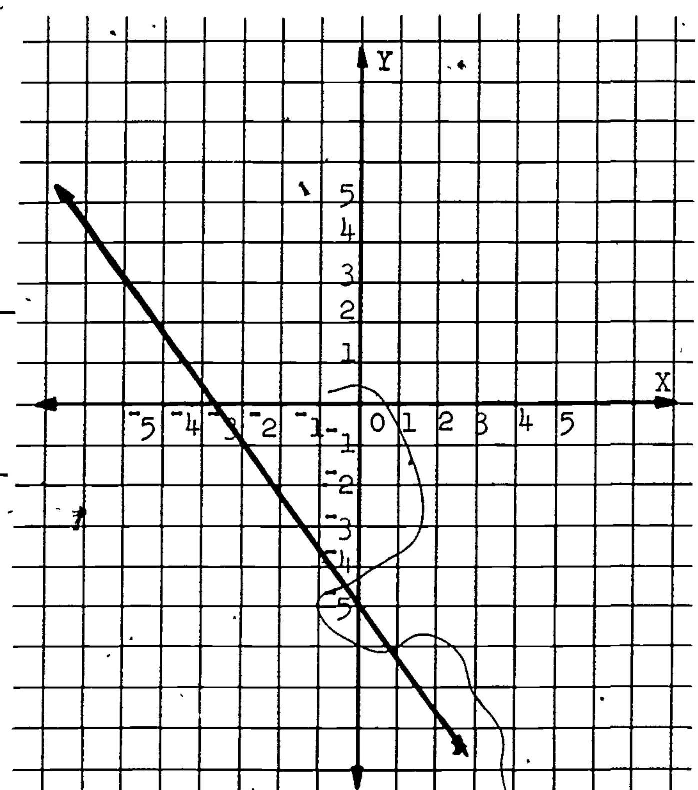
(c) Write the function for this line.

$f : x \rightarrow$ __________

(d) Write the equation of this line.

$y =$ __________

2.



(a) What is the slope of this line? __________

(b) What is the y-intercept of this line? __________

(c) Write the function for this line.

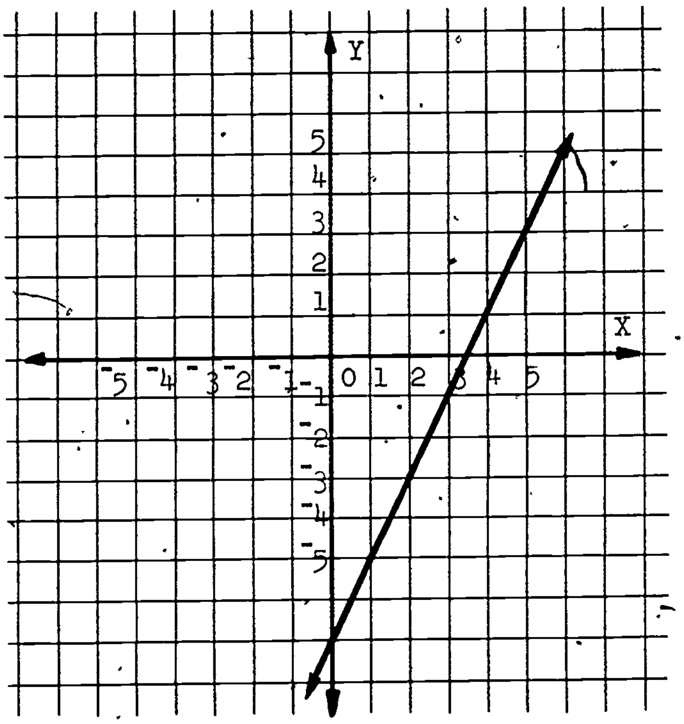
$f : x \rightarrow$ __________

(d) Write the equation of the line.

____

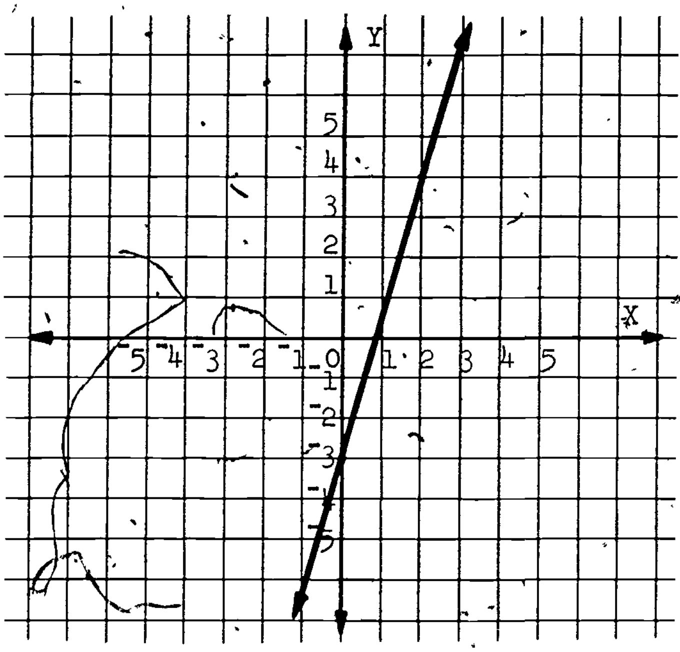
3. Write the equation of each of these lines.

(a)



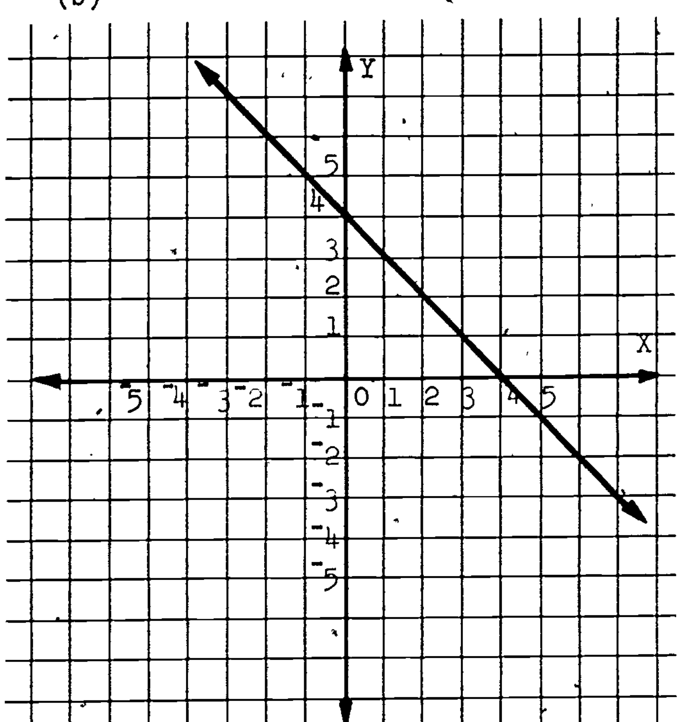
y = ____________

(c)



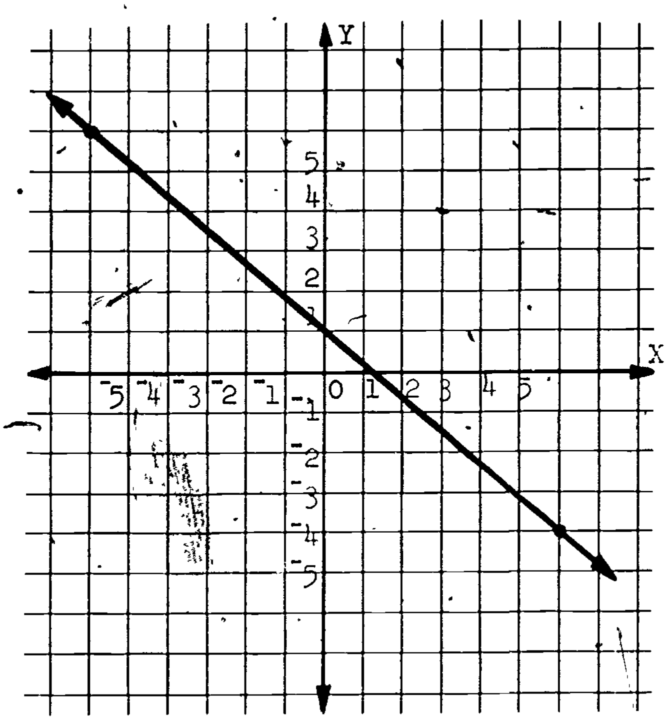
____

(b)



____

(d)



____

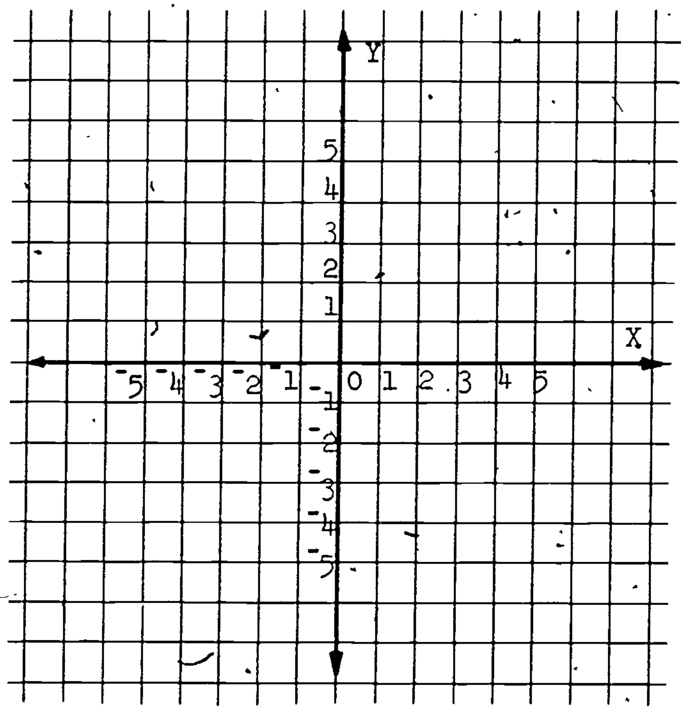
4. The equation of a line is

$$y = \frac{3}{5}x + 2.$$

(a) What is the slope of the line? ____________

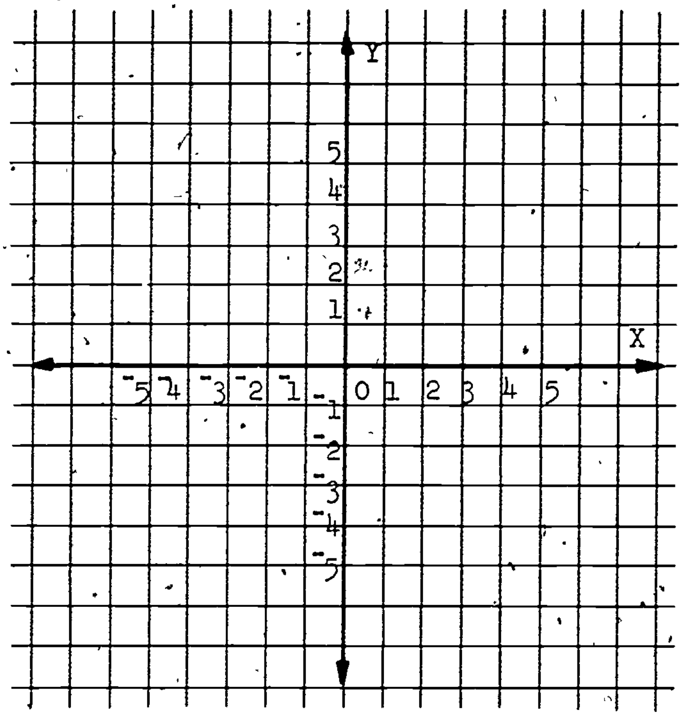
(b) What is the y-intercept of the line? ____________

(c) Draw the graph of the line without making a table of inputs and outputs.

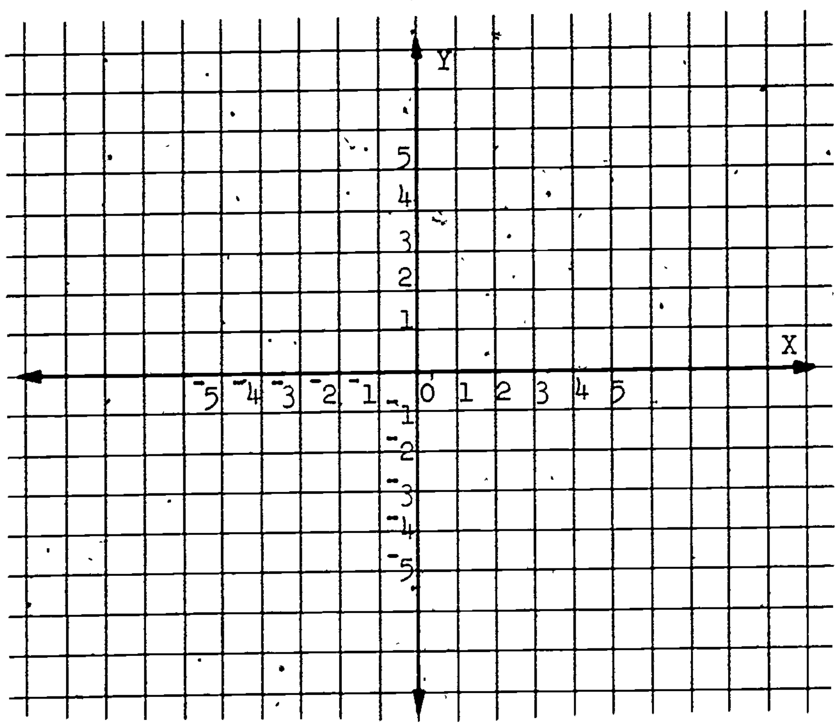


5. Draw the graph of each of these lines without making a table of inputs and outputs.

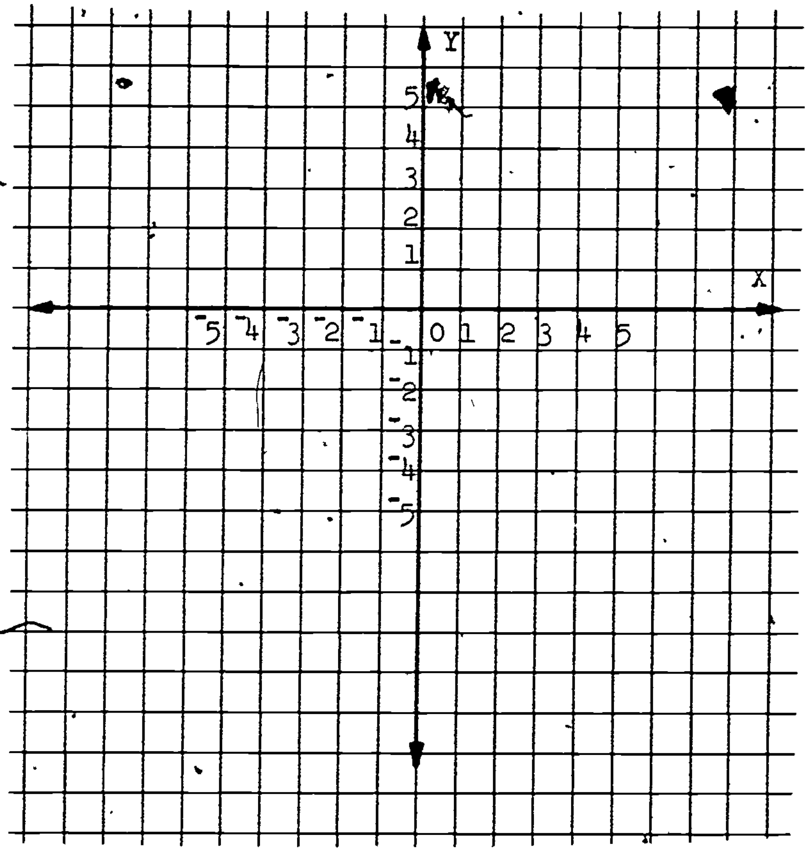
(a) $y = \frac{5}{4}x - 3$ (Hint. Write this as addition.) ____________



(b) $y = \frac{-5}{8}x + 1$



(c) $y = \frac{5}{7}x - 5$



Solution of Two Linear Equations

Here are some linear equations like the ones you graphed in the last lesson.

$$y = 3x + 4$$

$$y = \frac{^-3}{8}x + 7$$

$$y = {}^-12x - 3$$

Notice that these are different kinds of equations from the ones you studied in Chapter 17. They have two variables, x and y. These equations do not have a single solution. Instead they have <u>many</u> solutions.

The solutions of these equations are <u>pairs of numbers</u>. The pair of coordinates of every point on the line is a solution of the equation of that line.

You know that:

(1) if two lines are parallel they do not intersect

or (2) if they are not parallel they intersect in exactly one point.

If we have the equations of two lines that intersect then the pair of coordinates of the point of intersection is the <u>only</u> solution of the pair of equations.

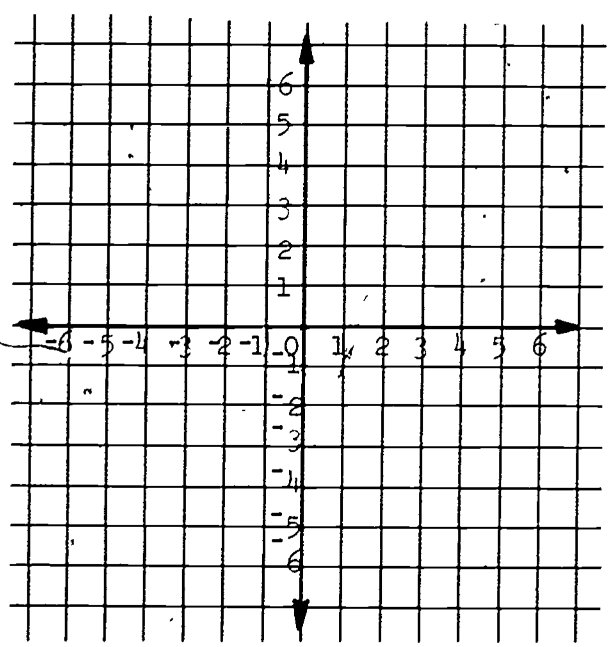
Class Discussion

1. Draw the graphs of these linear equations.

$$y = \frac{4}{3}x - 2$$

and

$$y = \frac{^-1}{3}x + 3$$



(a) What are the coordinates of the point of intersection of the pair of lines? (,) This is the solution of the pair of equations.

(b) What is the x-coordinate of the point of intersection?

____

(c) What is the y-coordinate of the point of intersection?

____

(d) Use these values of x and y to replace the variables in the first equation and do the arithmetic.

$$y = \frac{4}{3}x - 2$$

$$\underline{\quad} = \frac{4}{3} \cdot \underline{\quad} + {}^{-}2$$

$$\underline{\quad} = \underline{\qquad} + {}^{-}2$$

$$\underline{\quad} = \underline{\quad}$$

(e) Is (3,2) a solution of $y = \frac{4}{3}x - 2$? ________

(f) Use these same x and y values to replace the variables in the second equation and do the arithmetic.

$$y = \frac{{}^{-}1}{3}x + 3$$

$$\underline{\quad} = \frac{{}^{-}1}{3} \cdot \underline{\quad} + 3$$

$$\underline{\quad} = \underline{\qquad} + 3$$

$$\underline{\quad} = \underline{\quad}$$

(g) Is (3,2) a solution of $y = \frac{{}^{-}1}{3}x + 3$? ________

(h) Is (3,2) the only solution of this <u>pair</u> of equations? ________

$$y = \frac{4}{3}x - 2$$

and

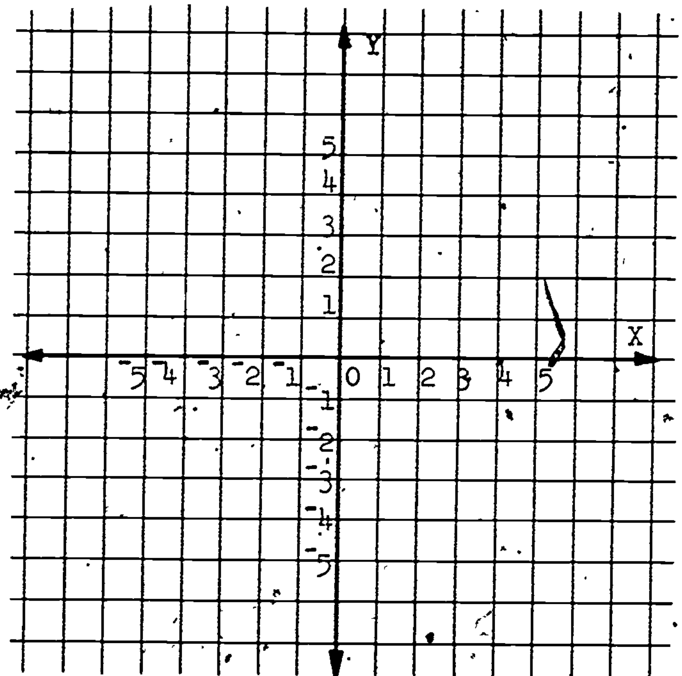
$$y = \frac{{}^{-}1}{3}x + 3$$

2. Draw the graphs of these linear equations.

$$y = {}^-3x + 0$$

and

$$y = x + 4$$



(a) What are the coordinates of the point of intersection of the pair of lines? (___ , ___) This is the solution of the pair of equations.

(b) Use the coordinates of the point of intersection to replace the variables in each equation and do the arithmetic.

$y = {}^-3x + 0$ and $y = x + 4$

___ $= {}^-3 \cdot$ ___ $+ 0$ ___ $=$ ___ $+ 4$

___ $=$ ___   ___ $=$ ___

(c) Is $({}^-1,3)$ the only solution of the pair of equations $y = {}^-3x + 0$ and $y = x + 4$? ________

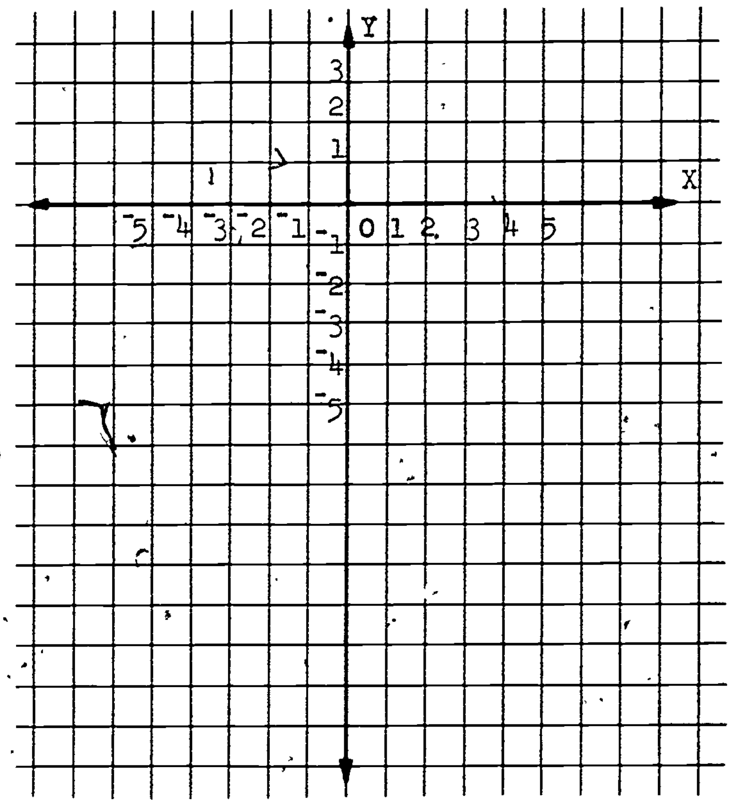
Exercises

1. (a) Find the solution of these two equations by graphing them and writing the coordinates of their intersection.

$$y = 3x - 7$$

and

$$y = \frac{{}^{-}1}{2}x + 0$$



Solution (,)

(b) Check your solution by replacing the variables in each equation with the coordinates of the point of intersection.

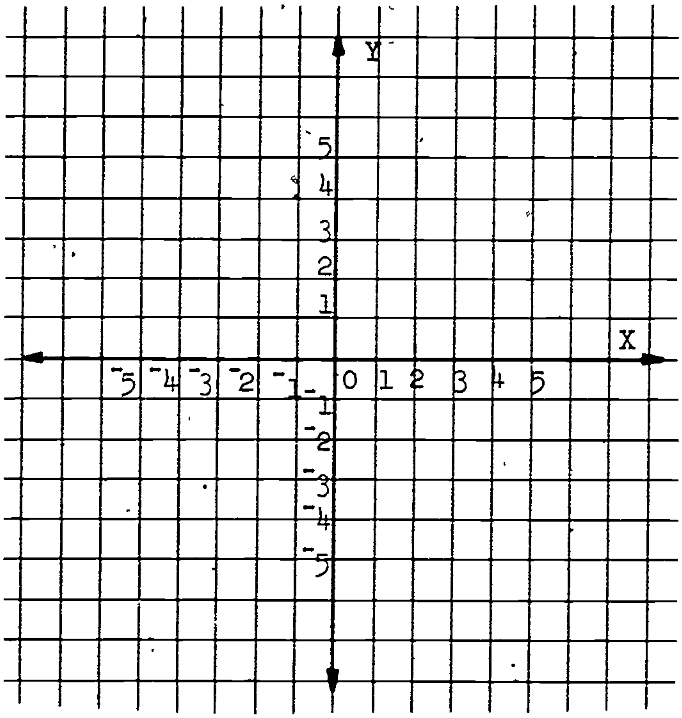
$y = 3x - 7$	$y = \frac{{}^{-}1}{2}x + 0$
___ = 3 · ___ - 7	___ = $\frac{{}^{-}1}{2}$ · ___ + 0
___ = _______ - 7	___ = ___
___ = ___	

2. (a) Find the solution of these two equations by graphing them and writing the coordinates of their intersection.

$$y = \frac{3}{2}x - 3$$

and

$$y = {}^{-}2x + 4$$



Solution (___ , ___)

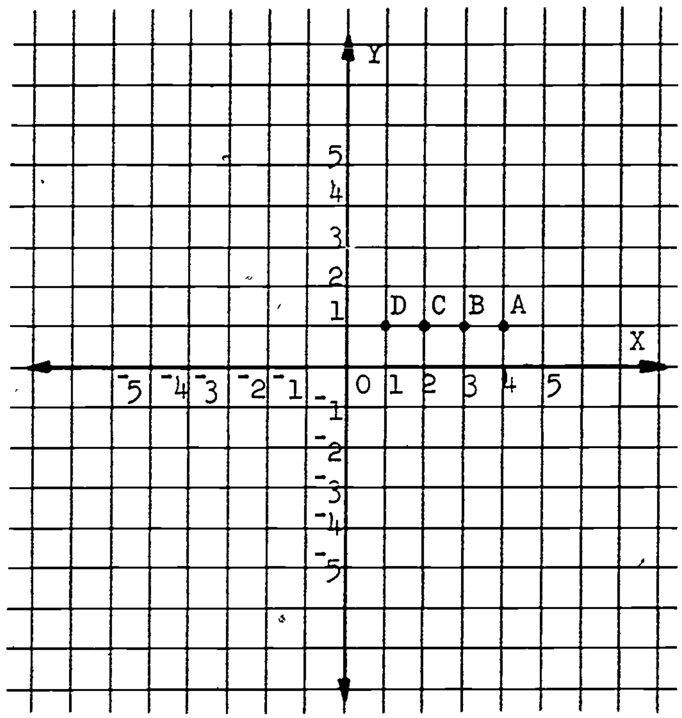
(b) Check your solution by replacing the variables in each equation with the coordinates of the point of intersection.

$y = \frac{3}{2}x - 3$	$y = {}^{-}2x + 4$
___ $= \frac{3}{2} \cdot$ ___ $+ {}^{-}3$	___ $= {}^{-}2 \cdot$ ___ $+ 4$
___ $=$ ______ $+ {}^{-}3$	___ $=$ ______ $+ 4$
___ $=$ ___	___ $=$ ___

Perpendicular Lines

You know that the axes of the coordinate plane are perpendicular to each other.

We have marked points A(4,1) ; B(3,1) ; C(2,1) ; D(1,1) ; on this graph.



Class Discussion

1. Place a piece of thin paper over the graph. Use a straightedge and trace the coordinate axes. Label your tracing of the X-axis ℓ_1 and your tracing of the Y-axis ℓ_2 .

2. Stick the needle point of your compass through the intersection of ℓ_1 and ℓ_2 and into the origin of the graph.

3. Turn your tracing about the origin so that ℓ_1 passes through point $A(4,1)$.

   (a) Is ℓ_1 still perpendicular to ℓ_2 ? ________

   (b) What is the slope of ℓ_1 ? ________

   (c) What is the slope of ℓ_2 ? ________

   (d) Multiply the slope of ℓ_1 by the slope of ℓ_2.

   ____ • ____ = ____

4. Now turn your tracing so that ℓ_1 passes through point $B(3,1)$.

   (a) Is $\ell_1 \perp \ell_2$? ________

   (b) What is the slope of ℓ_1 ? ________

   (c) What is the slope of ℓ_2 ? ________

   (d) Multiply the slope of ℓ_1 by the slope of ℓ_2.

   ____ • ____ = ____

   (e) Is the product of the slopes still the same as in problem 3 ? ________

5. Turn your tracing so that ℓ_1 passes through point $C(2,1)$.

   (a) Is $\ell_1 \perp \ell_2$? ________

   (b) What is the slope of ℓ_1 ? ________

   (c) What is the slope of ℓ_2 ? ________

   (d) Multiply the slope of ℓ_1 by the slope of ℓ_2.

   ____ • ____ = ____

   (e) Is the product of the slopes still the same? ________

6. Turn your tracing once more so that ℓ_1 passes through point $D(1,1)$.

(a) What is the slope of ℓ_1 ? ________

(b) What is the slope of ℓ_2 ? ________

(c) Multiply the slope of ℓ_1 times the slope of ℓ_2.

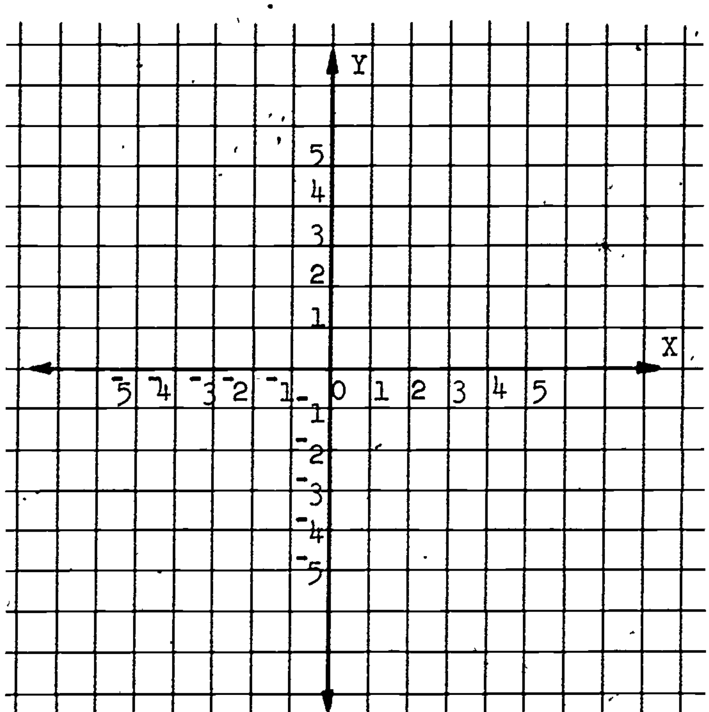
_____ $\cdot$ _____ = _____

(d) Is the product of the slopes still the same? ________

These two ideas go together:

> (1) If two lines are perpendicular then the product of their slopes is $^-1$.
>
> (2) If the product of the slopes of two lines is $^-1$ then the lines are perpendicular.

7. Draw the graph of the line $y = 3x + 0$. Label the line ℓ_1.



Suppose we want to write the equation of line ℓ_2 that has a y-intercept of 2 and that is perpendicular to ℓ_1.

(a) What is the slope of ℓ_1 ? ________

(b) What is the reciprocal of the slope of ℓ_1 ? ________

(c) What is the opposite of the reciprocal of the slope of ℓ_1 ?

____

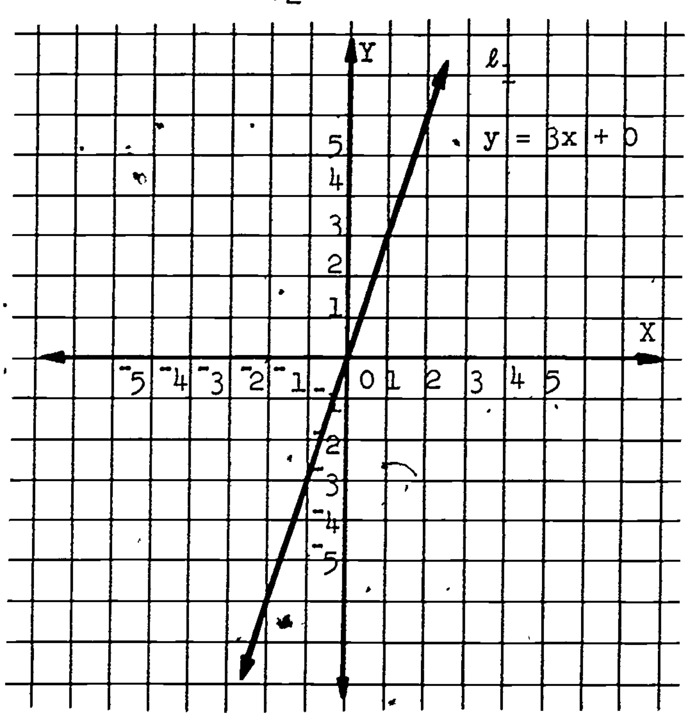
(d) If ℓ_2 has this slope will it be perpendicular to ℓ_1 ?

________ (Multiply these slopes and see if their product is $^-1$.)

(e) What is the equation of ℓ_2 ? ________________

The slope of ℓ_2 is the <u>opposite of the reciprocal</u> of the slope of ℓ_1.

(f) Draw the graph of ℓ_2.



(g) Does ℓ_2 look perpendicular to ℓ_1 ? ________

8. (a) What is the slope of the line $y = \frac{2}{3}x + 3$? ________

(b) What is the reciprocal of this slope? ________

(c) What is the opposite of the reciprocal of the slope? ________

(d) What is the equation of the line that has a y-intercept of -1 and is perpendicular to $y = \frac{2}{3}x + 3$? ____________

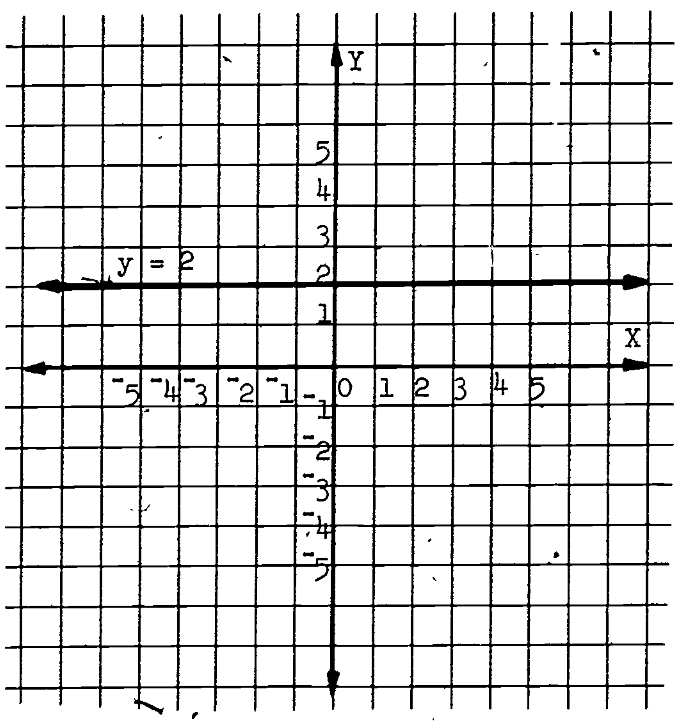
9. (a) What is the slope of $y = x - 2$? ________

(b) What is the reciprocal of this slope? ________ (What number multiplied by 1 equals 1 ?)

(c) What is the opposite of the reciprocal of the slope? ________

(d) What is the equation of the line that has a y-intercept of 5 and is perpendicular to $y = x - 2$? ____________

Here is the graph of $y = 2$. It is a horizontal line.

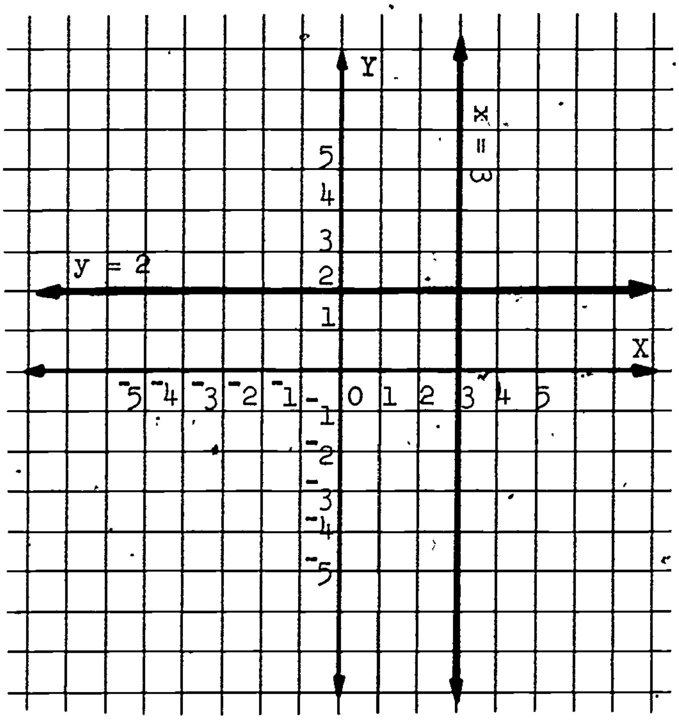


Start at (0,2) and count over 1 unit to the right. This is a run of 1. What is the rise? ________

$$\frac{\text{rise}}{\text{run}} = \frac{0}{1}$$

$\boxed{\text{The slope of a horizontal line is zero.}}$

Here is the graph of the line $x = 3$.



Is the line $x = 3$ a vertical line? ________

Is it perpendicular to the line $y = 2$? ________

But the slope of $y = 2$ is zero. Is there any number that you can multiply by zero and get $^-1$? ________

$\boxed{\text{There is no number we can give as the slope of a vertical line.}}$

It is meaningless to talk about the slope of a vertical line.

Exercises

1. Matching:

If you drew the lines for the equations below, you would find that each line under B is perpendicular to a line under A. Match the equations of the perpendicular lines.

A		B
$y = 5x + 7$	______	(a) $y = \frac{8}{5}x - \frac{1}{7}$
$y = \frac{^-1}{4}x + 2$	______	(b) $y = \frac{^-1}{5}x - 14$
$y = \frac{2}{3}x + 0$	______	(c) $y = \frac{^-7}{3}x + \frac{1}{2}$
$y = \frac{^-5}{8}x - \frac{4}{5}$	______	(d) $y = 4x - \frac{5}{6}$
$y = \frac{3}{7}x - \frac{1}{2}$	______	(e) $y = \frac{^-3}{2}x + \frac{2}{3}$

2. (a) What is the slope of the line $y = 5x - \frac{2}{3}$? ______

(b) What is the reciprocal of this slope? ______

(c) What is the opposite of the reciprocal? ______

(d) Write the equation of the line that has a y-intercept of 10 and is perpendicular to $y = 5x - \frac{2}{3}$. ______

3. Write the equation of the line that has a y-intercept of $^-7$ and is perpendicular to $y = \frac{4}{3}x + 8$. ______

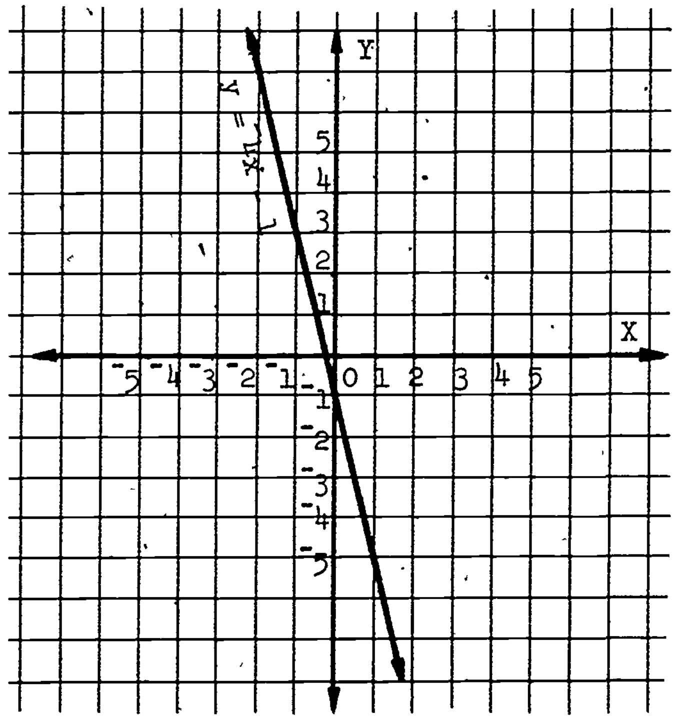
4. Write the equation of the line that has a y-intercept of $\frac{7}{8}$ and is perpendicular to $y = \frac{^-3}{5}x - \frac{1}{4}$. ______

5. Write the equation of the line that has a y-intercept of 14 and is perpendicular to $y = {^-5}x + \frac{2}{3}$. ______

Parallel Lines

Class Discussion

Here is the graph of $y = {}^-4x - 1$.



1. (a) What is the equation of the line ℓ_1 that has a y-intercept of 1 and that is perpendicular to $y = {}^-4x - 1$? __________

   (b) What is the equation of the line ℓ_2 that has a y-intercept of 2 and that is perpendicular to $y = {}^-4x - 1$? __________

   (c) Draw ℓ_1 and ℓ_2 on this graph.

   (d) If two lines are perpendicular to the same line are they parallel to each other? __________

   (e) Is $\ell_1 \parallel \ell_2$? __________

   (f) What is the slope of ℓ_1 ? __________

   (g) What is the slope of ℓ_2 ? __________

   (h) Are these slopes equal? __________

> If two lines are parallel they have the same slope,
>
> and
>
> if two lines have the same slope they are parallel.

Exercises

1. If you drew the lines for the equations below, you would find that each line under B is parallel to a line under A. Match the equations of the parallel lines.

A		B
$y = 3x - 2$	______	(a) $y = \frac{3}{4}x - 12$
$y = \frac{3}{4}x + 5$	______	(b) $y = \frac{1}{8}x + \frac{1}{2}$
$y = 17x + 6$	______	(c) $y = 3x + 0$
$y = \frac{1}{8}x - 7$	______	(d) $y = {}^-5x - \frac{3}{7}$
$y = {}^-5x + \frac{1}{6}$	______	(e) $y = 17x - 20$

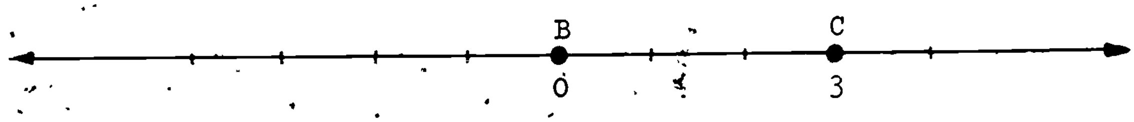
2. Write the equation of the line that has a y-intercept of 8 and that is parallel to $y = \frac{2}{5}x - 2$. ____________

3. Write the equation of the line that has a y-intercept of $^-5$ and is parallel to $y = 7x + 23$. ____________

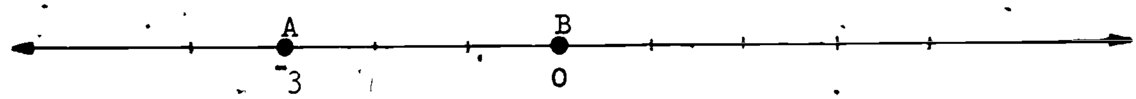
4. Write the equation of the line that has a y-intercept of $\frac{5}{8}$ and is parallel to $y = \frac{5}{2}x - \frac{3}{4}$. ____________

Absolute Value and Distance

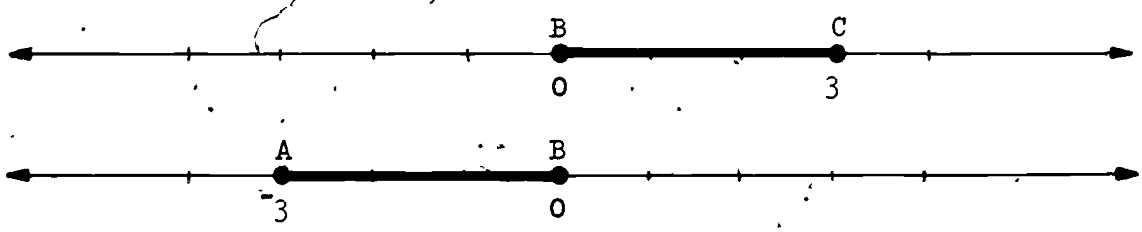
On the number line, you know that 3 names an integer that is three units to the right of zero,



and $^{-}3$ names an integer that is three units to the left of zero.



Point A and point C are both the same distance from point B . That is, $\overline{AB}$ and $\overline{CB}$ have the same length.



Since it doesn't make sense to talk about negative distances or negative lengths, we need some way to show that the distance between some number and zero on the number line is a positive number. We call this distance the absolute value of the number. So the absolute value of 3 is 3 and the absolute value of $^{-}3$ is 3 . This is the same thing as saying that $\overline{AB}$ and $\overline{BC}$ are both 3 units long.

Instead of writing "the absolute value of" every time we want to refer to a distance on the number line we put vertical bars on both sides of the number like this: $|^{-}3|$. This is read "the absolute value of $^{-}3$" . So

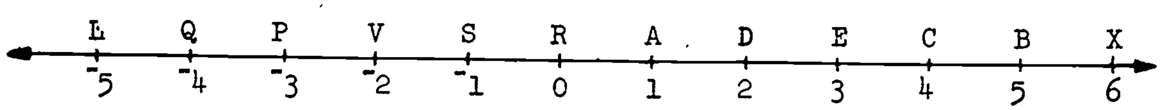
$$|^{-}3| = 3 \quad \text{and} \quad |3| = 3$$

Here are a few more examples.

$$|{}^-5| = 5 \quad \text{and} \quad |5| = 5$$
$$|{}^-17| = 17 \quad \text{and} \quad |17| = 7$$
$$|{}^-39| = 39 \quad \text{and} \quad |39| = 39$$

Class Discussion

1. Here is a number line with some of the points named by letters. Find the distance from the origin, R, to each of the following points. (Hint. Remember, distance is always positive.)



(a) D ______ (f) E ______

(b) V ______ (g) A ______

(c) B ______ (h) R ______

(d) L ______ (i) Q ______

(e) P ______ (j) C ______

2. What is the common name for each of the following?

(a) $|{}^-8| =$ ______ (f) $|{}^-6| =$ ______

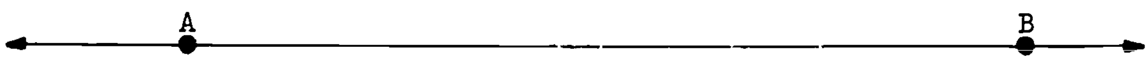
(b) $|7| =$ ______ (g) $|{}^-4| =$ ______

(c) $|{}^-9| =$ ______ (h) $|5| =$ ______

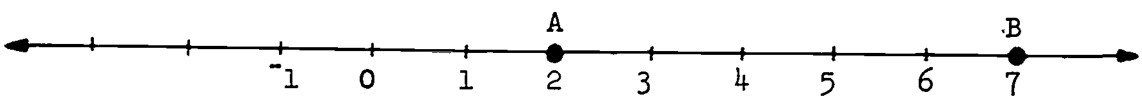
(d) $|2| =$ ______ (i) $|{}^-5| =$ ______

(e) $|{}^-2| =$ ______

The <u>distance</u> between two points on the number line is always shown by a positive number. If A and B are two points on the number line,



then the distance between A and B is the same as the distance between B and A. On this number line,



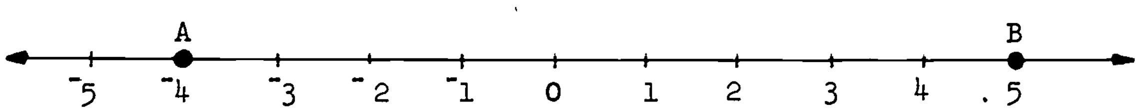
the distance between B and A is found by subtracting the coordinate of A from the coordinate of B .

$$7 - 2 = 5$$

The distance between A and B must also be 5 . If we subtract the coordinate of B from the coordinate of A the answer is a negative integer.

$$2 - 7 = {}^{-}5$$

But you know the distance must be a positive number. On this number line,



the distance between B and A is

$$\begin{aligned} 5 - ({}^{-}4) &= 5 + 4 \\ &= 9 \end{aligned}$$

So, the distance between A and B must also be 9 . But if we write $^{-}4 - 5$, we get

$$\begin{aligned} {}^{-}4 - 5 &= {}^{-}4 + {}^{-}5 \\ &= {}^{-}9 \end{aligned}$$

which is again a <u>negative</u> integer. You know it doesn't make sense to talk about a negative distance but if we use the idea of absolute value we can get away from this. Look at this example:

> If the distance from A to B is $^{-}4 - 5$ we can write this using the absolute value symbols, like this
>
> $$|^{-}4 - 5| .$$

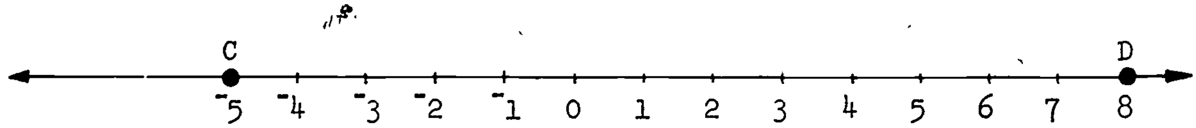
Then we do the arithmetic <u>inside</u> <u>the</u> <u>vertical</u> <u>bars</u> first. So,

$$\begin{aligned} |^{-}4 - 5| &= |^{-}4 + {}^{-}5| \\ &= |^{-}9| \\ &= 9 \end{aligned}$$

By doing it this way, we always get a positive number for the distance.

Here is one more example:

Find the distance between C and D .



We can find this distance in two ways.

Like this:

$$\begin{aligned} |8 - {}^{-}5| &= |8 + 5| \\ &= |13| \\ &= 13 , \end{aligned}$$

or like this:

$$\begin{aligned} |^{-}5 - 8| &= |^{-}5 + {}^{-}8| \\ &= |^{-}13| \\ &= 13 \end{aligned}$$

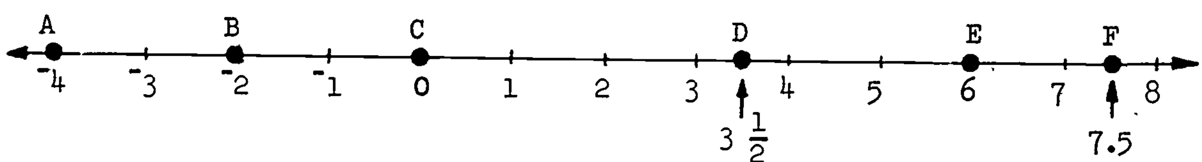
You can see that as long as you do the arithmetic inside the absolute value bars, it makes no difference whether the problem is $|8 - {}^{-}5|$ or $|^{-}5 - 8|$. The answer will always be positive.

Exercises

1. Find the value for each of the following:

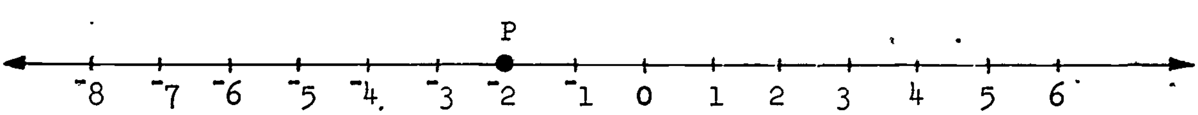
(a) $\lvert 8 \rvert =$ ______	(f) $\lvert 18 - 5 \rvert =$ ______
(b) $\lvert ^-10 \rvert =$ ______	(g) $\lvert 3 - 7 \rvert =$ ______
(c) $\lvert 23 - 2 \rvert =$ ______	(h) $\lvert 7 - 3 \rvert =$ ______
(d) $\lvert 2 - 23 \rvert =$ ______	(i) $\lvert 3\frac{1}{2} - 7\frac{1}{2} \rvert =$ ______
(e) $\lvert 5 - 18 \rvert =$ ______	(j) $\lvert 7\frac{1}{2} - 3\frac{1}{2} \rvert =$ ______

2. Find the distances between the following points marked on this number line.



(a) A and C ______	(f) A and F ______
(b) B and C ______	(g) B and D ______
(c) C and D ______	(h) F and E ______
(d) E and C ______	(i) C and B ______
(e) B and E ______	(j) F and A ______

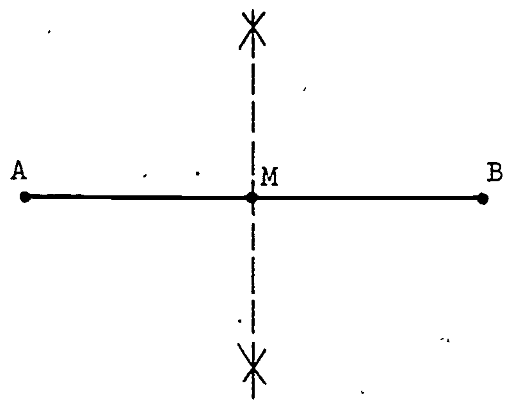
3. (a) On this number line, locate the points A and B, so that the distance between A and P is 5 and the distance between B and P is 5.



(b) What is the distance between A and B ? ______

The Coordinate of the Midpoint of a Segment

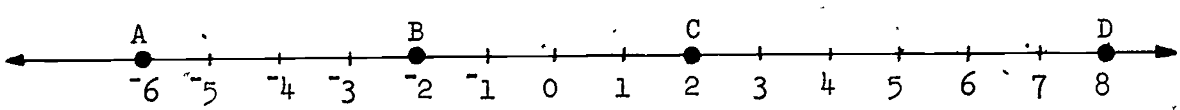
In the chapter on congruence you learned to bisect a line segment.



The point of bisection, M, is called the midpoint of $\overline{AB}$. Now you will learn how to find the coordinate of the midpoint of a segment.

Class Discussion

Points A, B, C, and D are shown on the number line below. Use this number line to answer the questions that follow.



1. (a) What is the coordinate of the point half-way between C and D? ______ This is the coordinate of the midpoint of $\overline{CD}$.

   (b) What is the coordinate of C? ______

   (c) What is the coordinate of D? ______

   (d) $\frac{2 + 8}{2} =$ ______

2. (a) What is the coordinate of the point <u>half-way</u> between A and B ? ______ This is the coordinate of the midpoint of $\overline{AB}$.

   (b) What is the coordinate of A ? ______

   (c) What is the coordinate of B ? ______

   (d) $\frac{^-6 + ^-2}{2} =$ ______

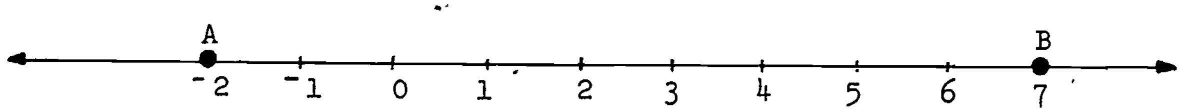
3. (a) What is the coordinate of the midpoint of $\overline{AD}$? ______

   (b) What is the coordinate of A ? ______

   (c) What is the coordinate of D ? ______

   (d) $\frac{^-6 + 8}{2} =$ ______

4. On this number line:

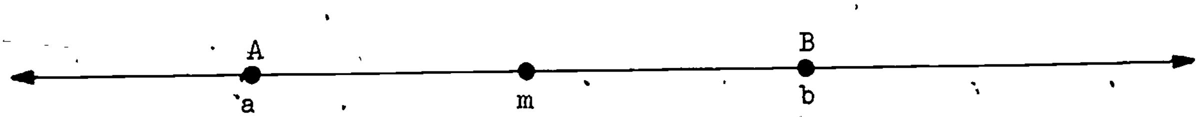


   (a) The coordinate of A is ______ .

   (b) The coordinate of B is ______ .

   (c) The coordinate of the midpoint is $\frac{\quad + \quad}{2}$ or ______ .

5. Suppose a and b are the coordinates of A and B on the number line (a and b are <u>numbers</u>).

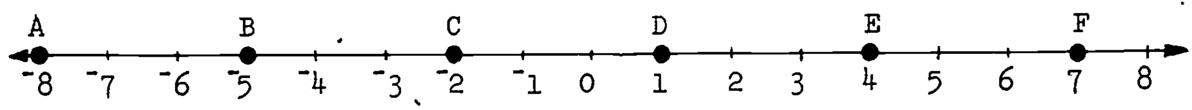


Write a formula for finding the coordinate of the midpoint m . (<u>Hint</u>. Look at your answers to part (d) in Problems 1, 2, and 3.)

$m = \frac{\quad + \quad}{\quad}$

Exercises

1. Find the coordinate of the midpoint of the following line segments.



<u>Example.</u> The coordinate of the midpoint of $\overline{AC}$ is

$\frac{^-8 + ^-2}{2} = \frac{^-10}{2}$ or $^-5$.

(a) The coordinate of the midpoint of $\overline{DF}$ is

$\frac{\quad + \quad}{\quad} = \frac{\quad}{\quad}$ or ______ .

(b) The coordinate of the midpoint of $\overline{CE}$ is

$\frac{\quad + \quad}{\quad} = \frac{\quad}{\quad}$ or ______ .

(c) The coordinate of the midpoint of $\overline{AD}$ is

$\frac{\quad + \quad}{\quad} = \frac{\quad}{\quad}$ = ______ .

(d) The coordinate of the midpoint of $\overline{BF}$ is

$\frac{\quad + \quad}{\quad} = \frac{\quad}{\quad}$ = ______ .

(e) The coordinate of the midpoint of $\overline{AF}$ is

$\frac{\quad + \quad}{\quad} = \frac{\quad}{\quad}$ = ______ .

2. Use the midpoint formula $\frac{a + b}{2} = m$ to find the coordinate of the midpoint of a segment whose endpoints have coordinates a and b .

<u>Example.</u> $a = {}^-3$, $b = 9$

$$m = \frac{a + b}{2}$$

$$= \frac{{}^-3 + 9}{2}$$

$$= \frac{6}{2}$$

$$m = 3$$

(a) $a = 0$, $b = {}^-10$

m = ______

= ______

= ______

(b) $a = {}^-12$, $b = {}^-4$

m = ______

= ______

= ______

(c) $a = 12$, $b = 18$

m = ______

= ______

= ______

(d) $a = {}^-1$, $b = {}^-8$

m = ______

= ______

= ______

(e) $a = {}^-7$, $b = 16$

m = ______

= ______

= ______

(f) $a = {}^-7$, $b = {}^-16$

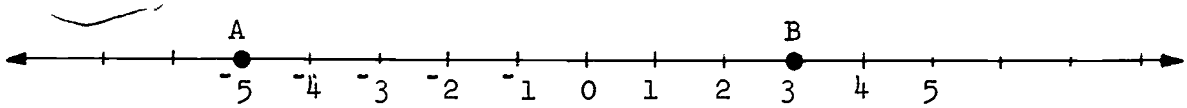
m = ______

= ______

= ______

<u>Distance Between Two Points in a Plane, Part I.</u>

You know how to find the distance between two points on the <u>number line</u> when you know the coordinates of the points. For example, on this number line



the distance between A and B is

$$\begin{aligned} |{}^-5 - 3| &= |{}^-5 + {}^-3| \\ &= |{}^-8| \\ &= 8 . \end{aligned}$$

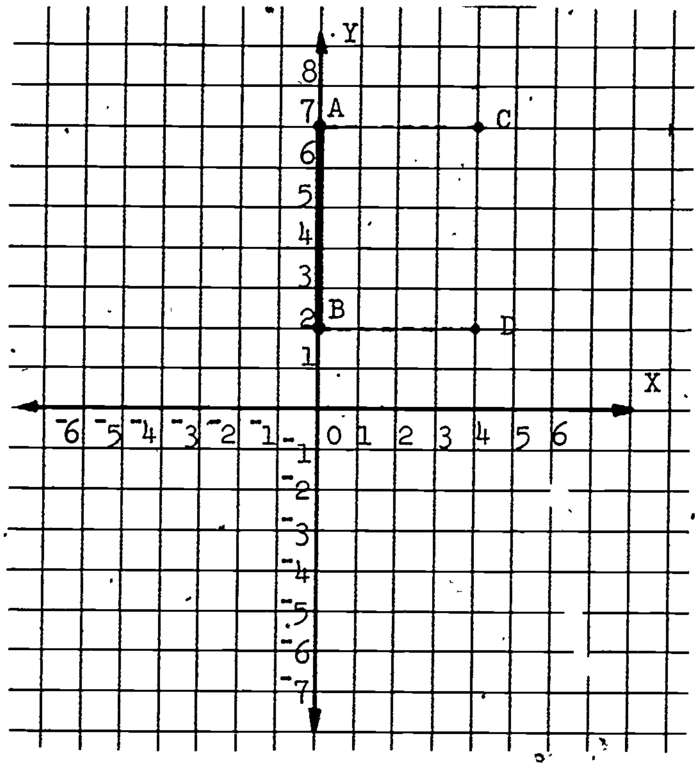
If you go the other way then the distance between B and A is

$$\begin{aligned} |3 - {}^-5| &= |3 + 5| \\ &= |8| \\ &= 8 . \end{aligned}$$

We can find the distance between two points in the <u>coordinate plane</u> when we know the coordinates of the points. First we will talk about the distance between two points that are on either horizontal or vertical lines but are not on the axes.

Class Discussion

1.



(a) What are the coordinates of point C ? (,)

(b) What are the coordinates of point D ? (,)

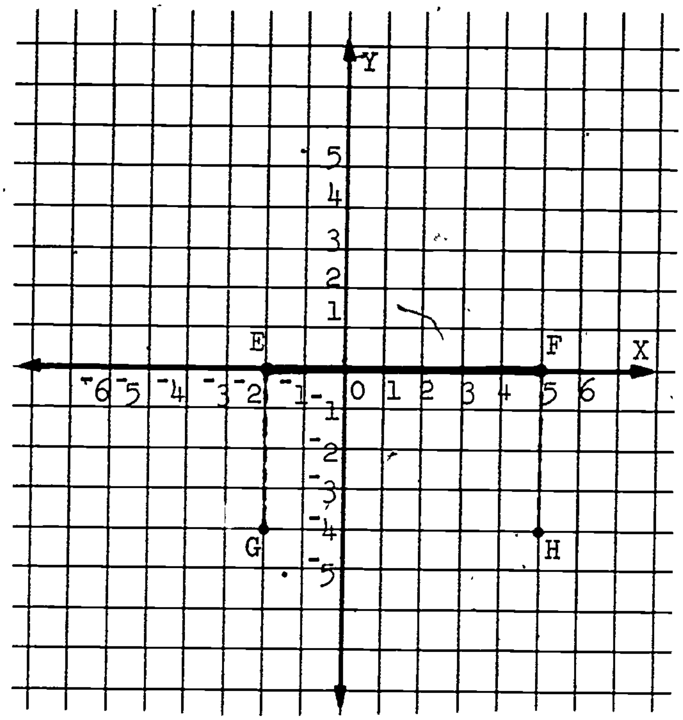
(c) What is the length of $\overline{AB}$? ______

(d) Is the y-coordinate of C the same as the y-coordinate of A ? ______

(e) Is the y-coordinate of D the same as the y-coordinate of B ? ______

(f) What is the length of $\overline{CD}$? ______

2.



(a) What are the coordinates of point G ? (,)

(b) What are the coordinates of point H ? (,)

(c) What is the length of $\overline{EF}$? ________

(d) Is the x-coordinate of E the same as the x-coordinate of G ? ________

(e) Is the x-coordinate of F the same as the x-coordinate of H ? ________

(f) What is the length of $\overline{GH}$? ________

To find the distance between two points that lie on a line parallel to the X-axis:

<u>Find the absolute value of the difference of the x-coordinates.</u>

To find the distance between two points that lie on a line parallel to the Y-axis:

<u>Find the absolute value of the difference of the y-coordinates.</u>

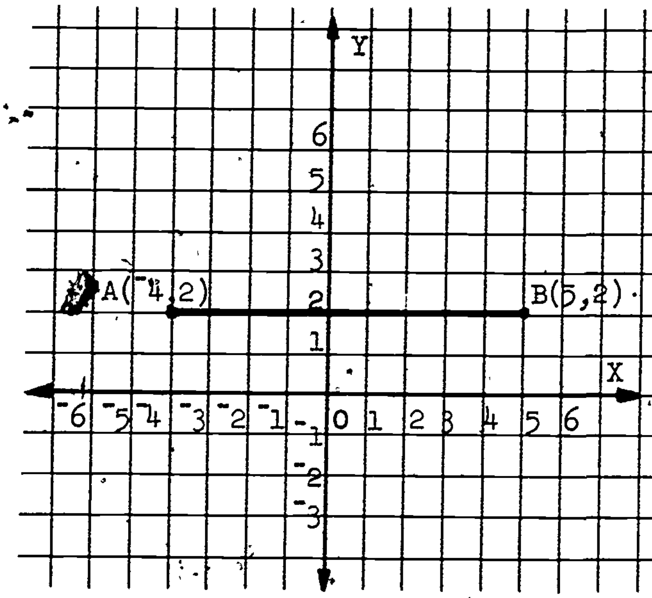
Example 1.

Find the distance between two points A and B whose coordinates are $A({}^-4,2)$ and $B(5,2)$.

Solution.

$$|{}^-4 - 5| = |{}^-4 + {}^-5|$$
$$= |{}^-9|$$
$$= 9$$

The length of $\overline{AB}$ is 9.



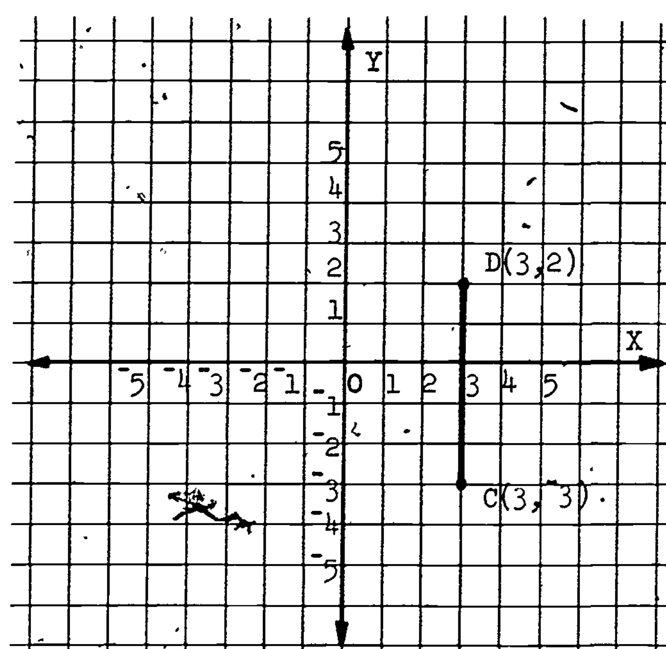
Example 2.

Find the distance between two points, C and D whose coordinates are $C(3,{}^-3)$ and $D(3,2)$.

Solution.

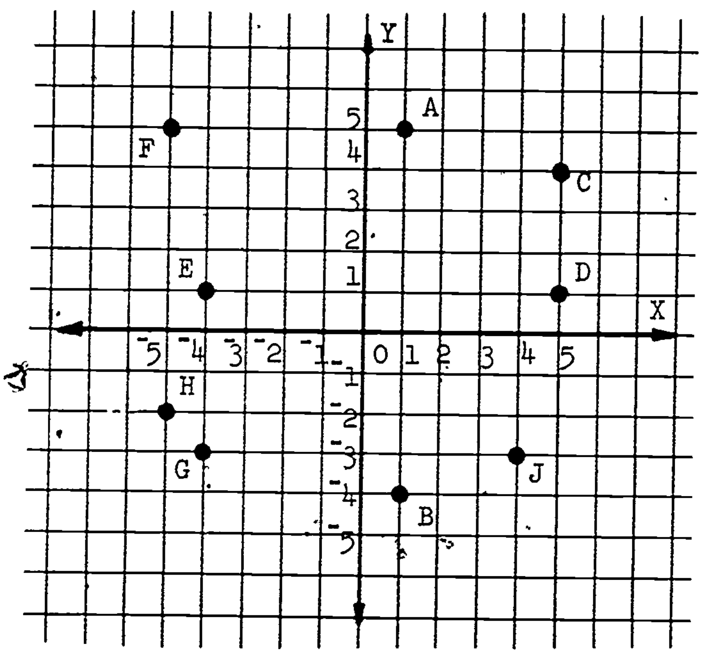
$$|{}^-3 - 2| = |{}^-3 + {}^-2|$$
$$= |{}^-5|$$
$$= 5$$

The length of $\overline{CD}$ is 5.



Exercises

1. On this coordinate plane some points have been located. Find the length of the following line segments.



(a) $\overline{AB}$ ________

(b) $\overline{CD}$ ________

(c) $\overline{DE}$ ________

(d) $\overline{FH}$ ________

(e) $\overline{GJ}$ ________

(f) $\overline{EG}$ ________

(g) $\overline{FA}$ ________

You can find distance even when the points are not <u>shown</u> on a coordinate plane.

2. Find the distance between two points, A and B, whose coordinates are A(2,5) and B(2,$^-3$). ________

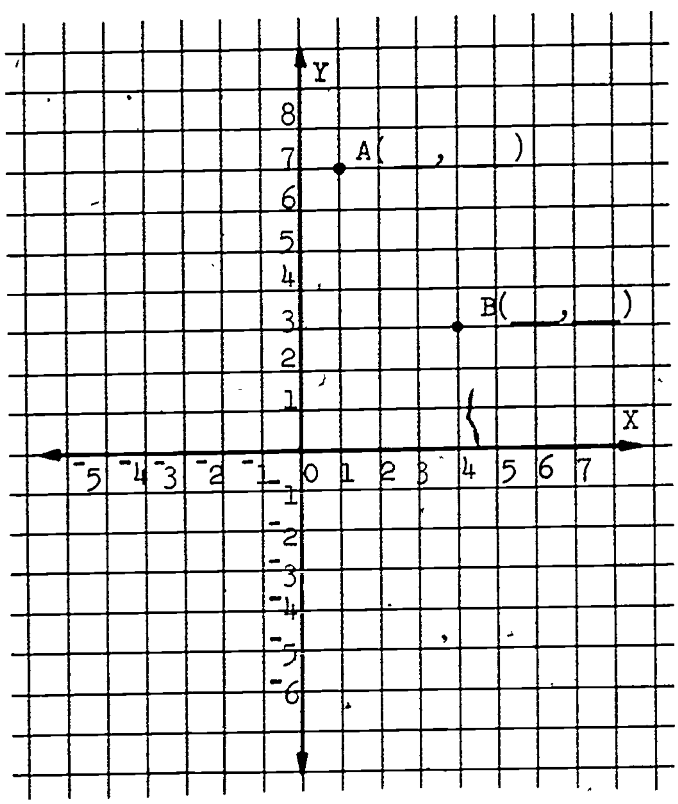
3. Find the distance between two points, C and D, whose coordinates are C($^-5$,$^-4$) and D(6,$^-4$). ________

Distance Between Two Points __ a Plane, Part II.

In the last lesson you learned how to find the distance between two points in the coordinate plane if the two points are on a line parallel to either the X-axis or the Y-axis. Now you are going to learn how to find the distance between <u>any</u> two points in the coordinate plane.

Class Discussion

On the coordinate plane below, we have located two points A and B .



1. Write the coordinates of points A and B in the blanks beside these points.

2. Use your straightedge and draw $\overline{AB}$.

3. Is $\overline{AB}$ on a line parallel to either axis? ________

4. Use your straightedge and draw the vertical line that passes through point A .

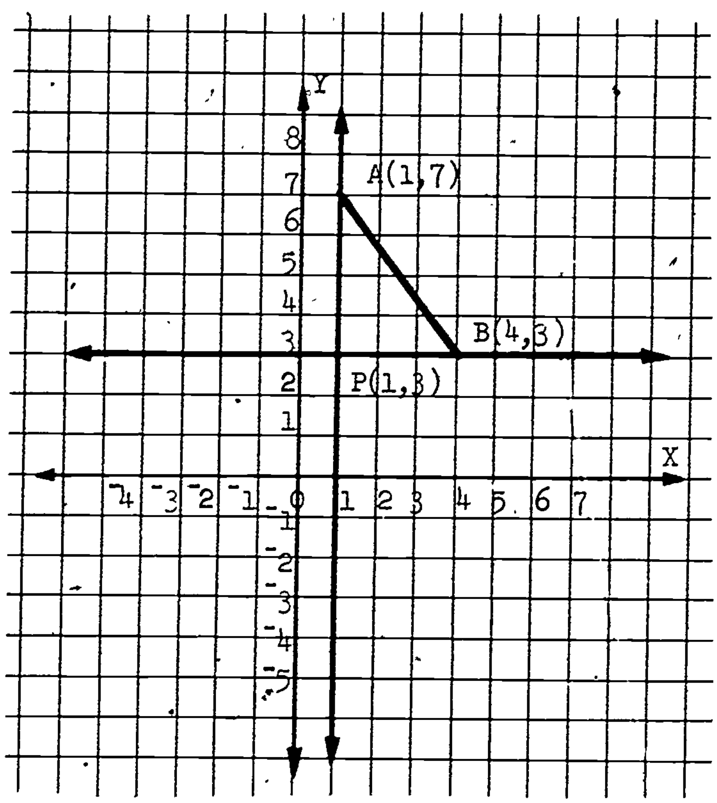
5. Use your straightedge and draw the horizontal line that passes through point B .

6. Do the two lines you just drew intersect? ______

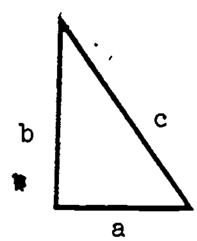
7. Label the point of intersection P . What are the coordinates of point P ? (,)

8. What kind of a figure is formed by $\overline{AP}$, $\overline{PB}$, and $\overline{AB}$? ______

If you followed the above directions correctly your figure should look like this.



You know that the vertical line through point A and the horizontal line through point B are perpendicular to each other at point P . So, the angle at P is a right angle, $\triangle APB$ is a <u>right triangle</u>, and $\overline{AB}$ is the <u>hypotenuse</u> of this triangle. To find the length of $\overline{AB}$ you use the Pythagorean Property. That is, for all right triangles



if <u>a</u> is the length of one leg of the triangle, and <u>b</u> is the length of the other leg of the triangle, then the square of the length of the hypotenuse, <u>c</u> , is found by the formula

$$c^2 = a^2 + b^2 , \text{ so}$$

$$c = \sqrt{a^2 + b^2} .$$

9. For the right triangle $\triangle APB$:

   (a) What is the length of $\overline{AP}$? ________

   (b) What is the length of $\overline{AP}$, squared? ________

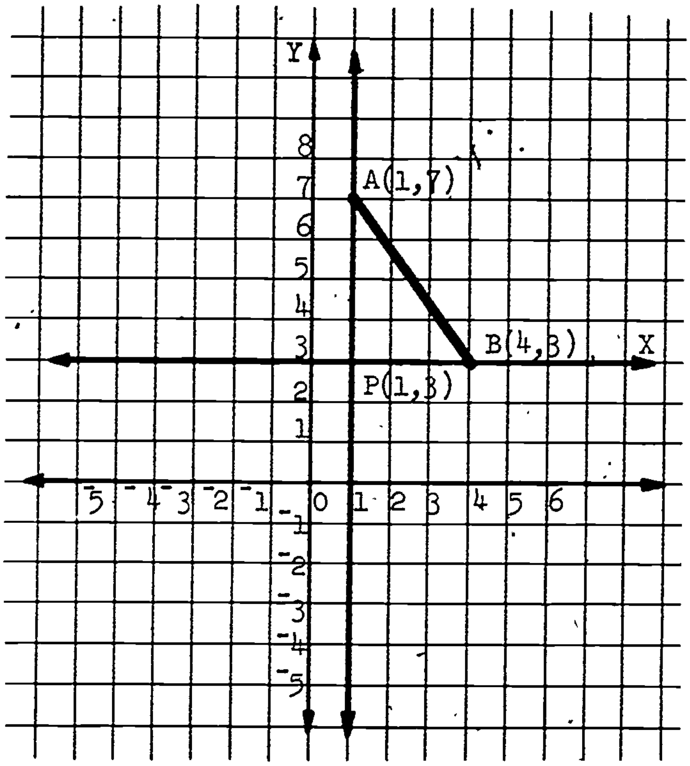
   (c) What is the length of $\overline{BP}$? ________

   (d) What is the length of $\overline{BP}$, squared ? ________

   (e) What is the sum? (length of $\overline{AP}^2$ + length of $\overline{BP}^2$) ________

   (f) What is the square root of the number you got for an answer to part (e)? ________ <u>This is the length of</u> $\overline{AB}$.

We will summarize what we just did.



(1) The length of $\overline{AP}$ is $|7 - 3|$ or <u>4</u> .

(2) The length of $\overline{BP}$ is $|4 - 1|$ or <u>3</u> .

(3) The length of $\overline{AB} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

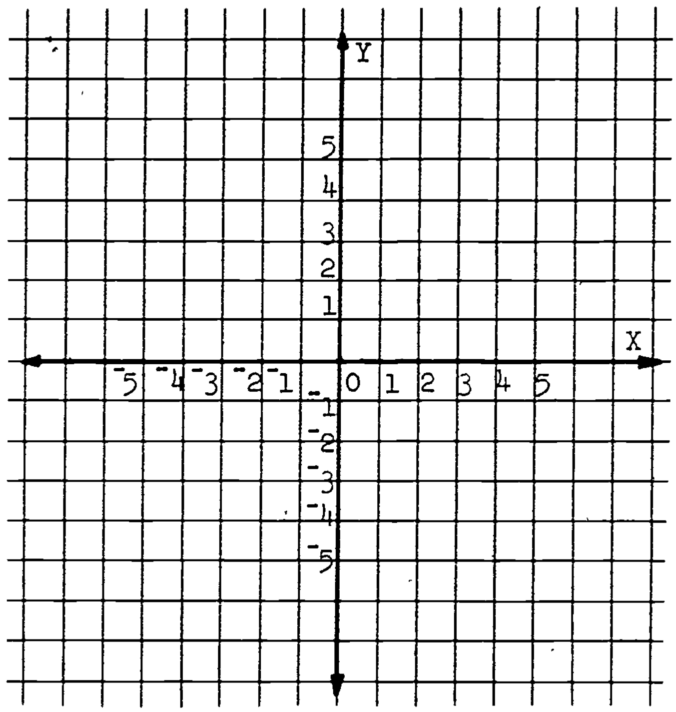
$= \sqrt{25}$

$= 5$

Exercises

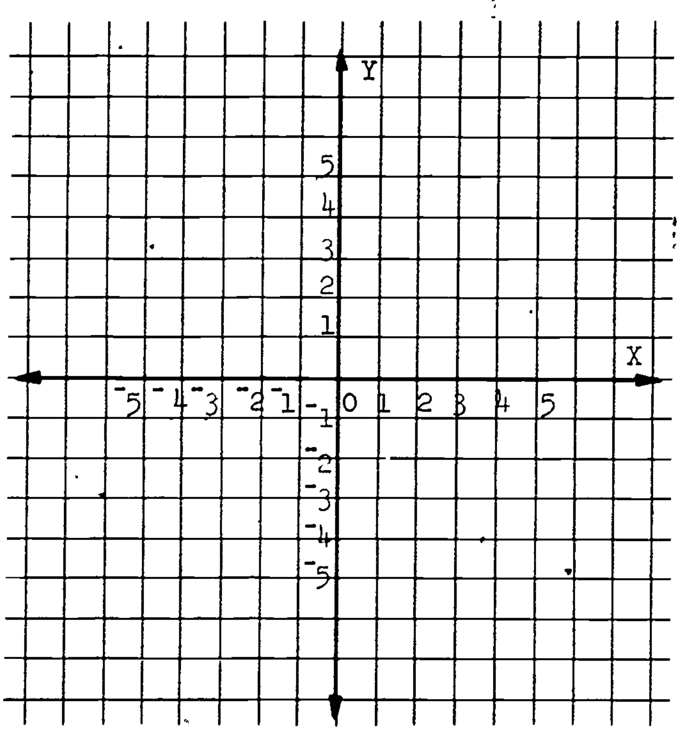
On these coordinate planes, plot the points A and B. Draw the vertical line through A and the horizontal line through B and locate point P. Then find the length of $\overline{AB}$.

1. $A(^{-}2, ^{-}2)$, $B(1, ^{-}6)$



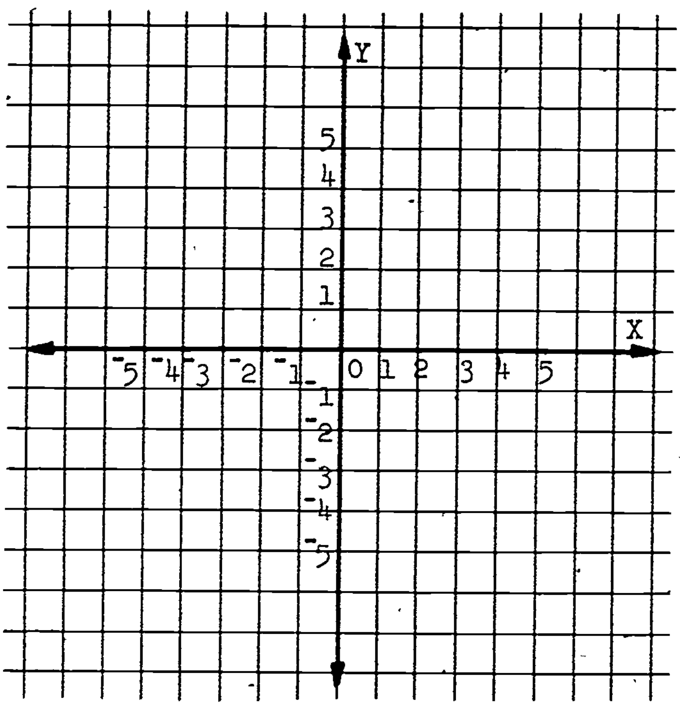
Length of $\overline{AB}$ = ______

2. $A(^{-}4, 3)$, $B(4, ^{-}3)$



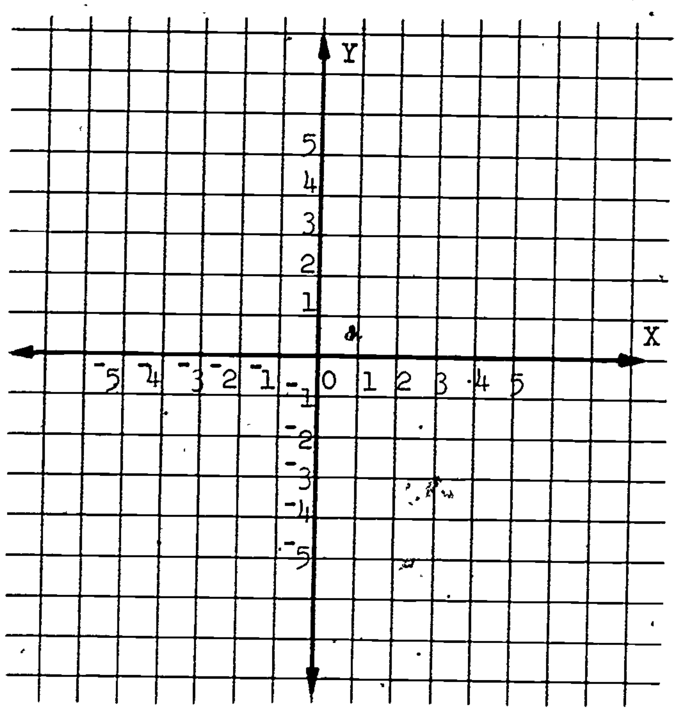
Length of $\overline{AB}$ = ______

3. $A(^-6,7)$, $B(^-1,^-5)$



Length of $\overline{AB}$ = ______

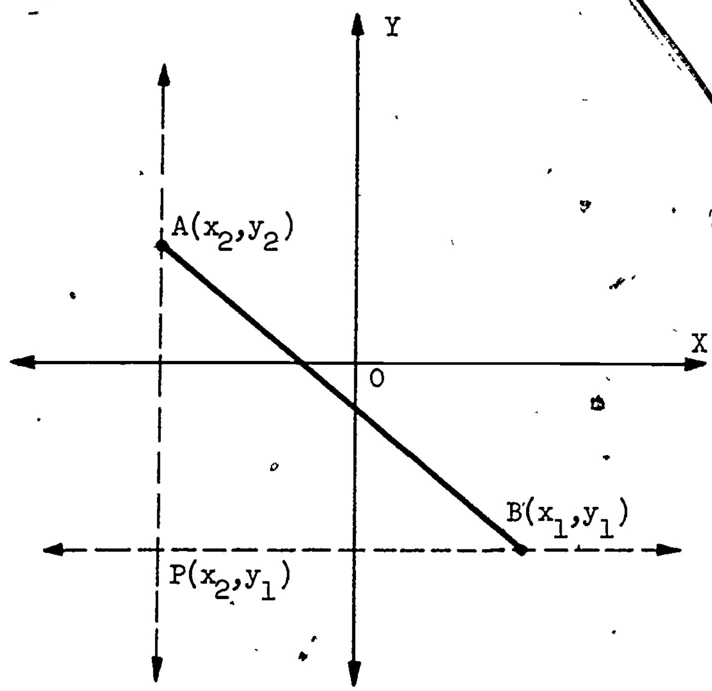
4. $A(3,4)$, $B(6,^-2)$



Length of $\overline{AB}$ = ______

SUPER BRAINBOOSTER.

5. Suppose we have a point A whose coordinates are (x_2,y_2). (x_2 and y_2 are numbers) and a point B whose coordinates are (x_1,y_1). (x_1 and y_1 are numbers different from x_2 and y_2).



Then the coordinates of point P are (x_2,y_1).

Use the absolute value symbol, what you know about finding the distance between two points on a line parallel to either the X-axis or Y-axis, and the Pythagorean Property to see if you can discover how we got this formula for finding the length of $\overline{AB}$.

$$\text{Length of } \overline{AB} = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

Pre-Test Exercises

These exercises are like the problems you will have on the chapter test. If you don't know how to do them, read the section again. If you still don't understand, ask your teacher.

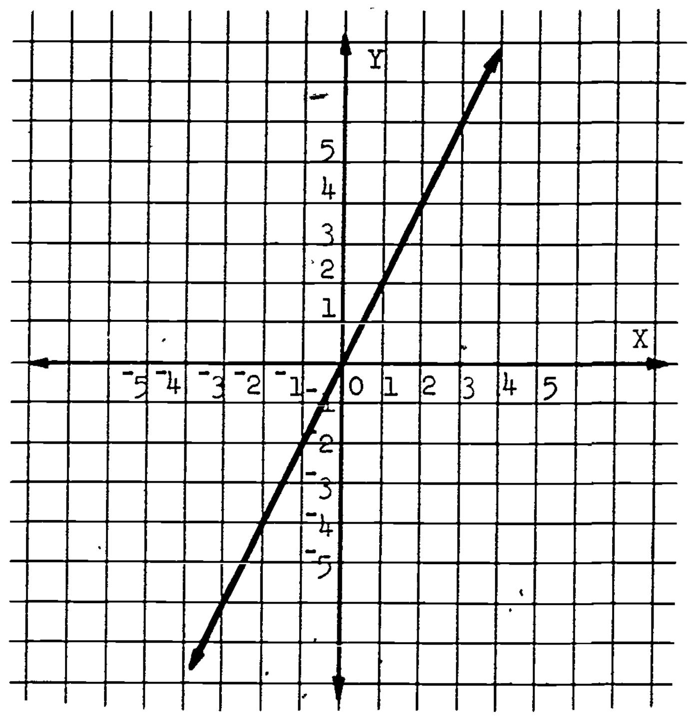
1. (Section 18-1)

For this graph:

(a) What is the rise of the line? ______

(b) What is the run of the line? ______

(c) What is the slope of the line? ______



2. (Section 18-2)

(a) Lines that have a positive slope lean to your ________ .

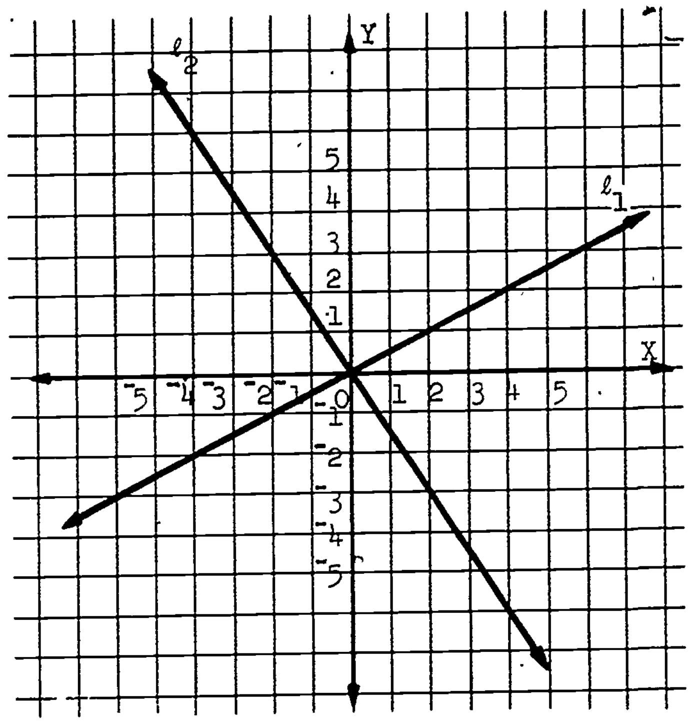
(b) Lines that have a negative slope lean to your ________ .

3. (Section 18-2)

Here are two lines, ℓ_1 and ℓ_2.

(a) What is the slope of ℓ_1 ? ______

(b) What is the slope of ℓ_2 ? ______

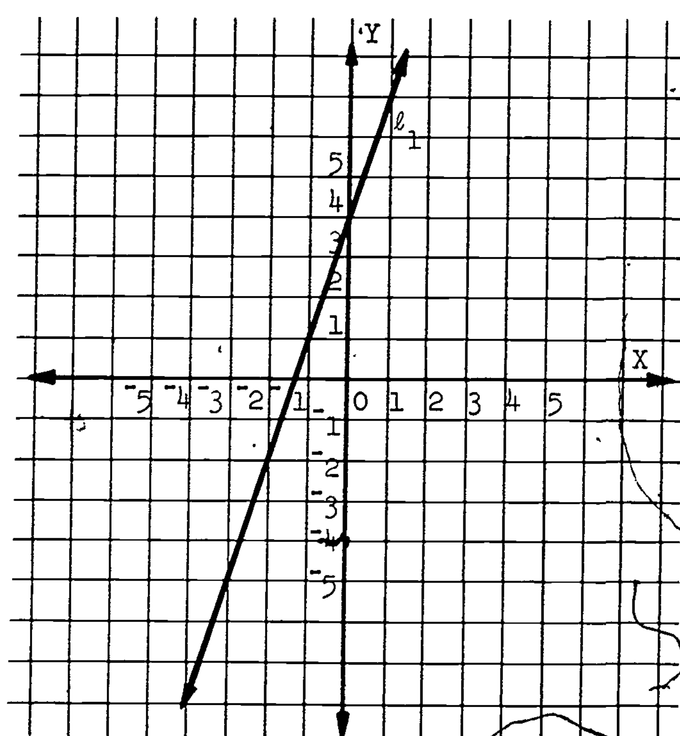


4. (Section 18-3)

Here is the graph of ℓ_1.

(a) What is the y-intercept of ℓ_1 ? ______

(b) What is the slope of ℓ_1 ? ______



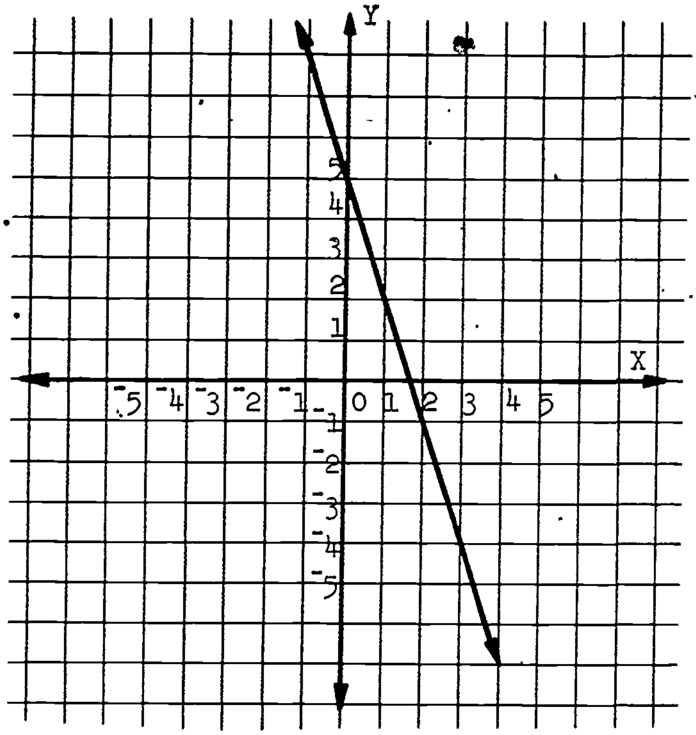
5. (Section 18-3)

Write the slope, the y-intercept, and the function for this line.

(a) Slope is ________ .

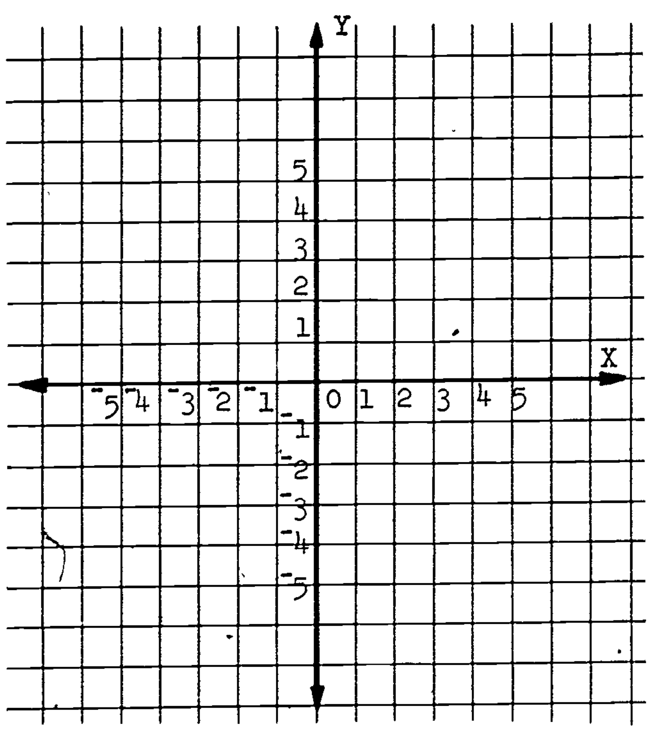
(b) y-intercept is ________ .

(c) $f : x \rightarrow$ ____________ .



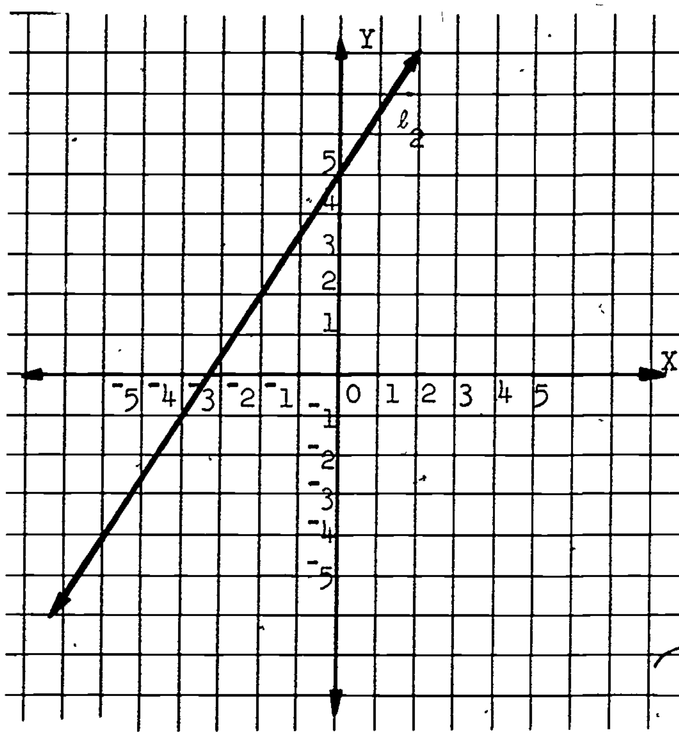
6. (Section 18-3)

Without making a table of inputs and outputs, draw the graph of $f : x \rightarrow 3x + {}^{-}2$.



7. (Section 18-4)

Here is the graph of ℓ_2 .



(a) What is the slope of this line? ________

(b) What is the y-intercept of this line? ________

(c) Write the function for this line.

$f : x_v \rightarrow$ ______________

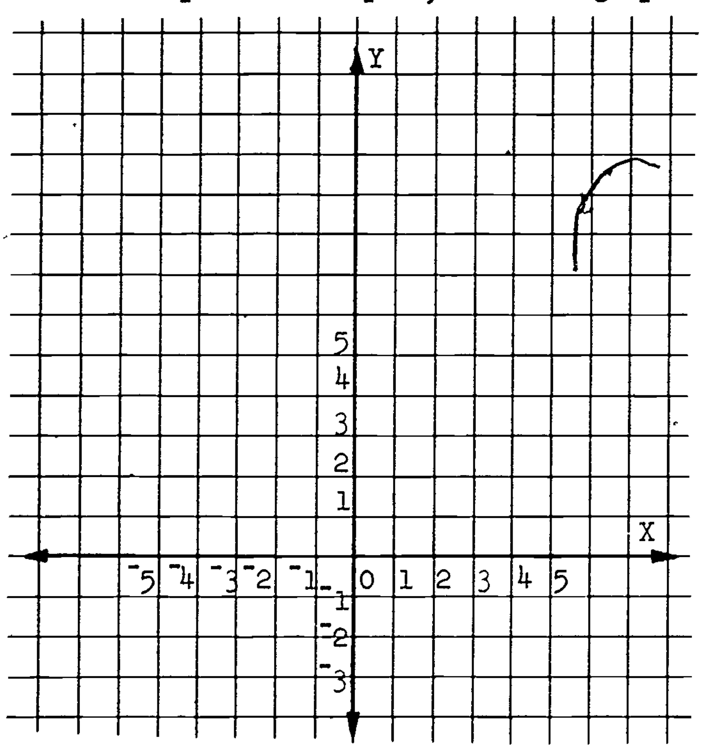
(d) Write the equation of this line.

y = ______________

8. (Section 18-4)

Without making a table of inputs and outputs, draw the graph of

$y = \frac{3}{4}x + 7$.



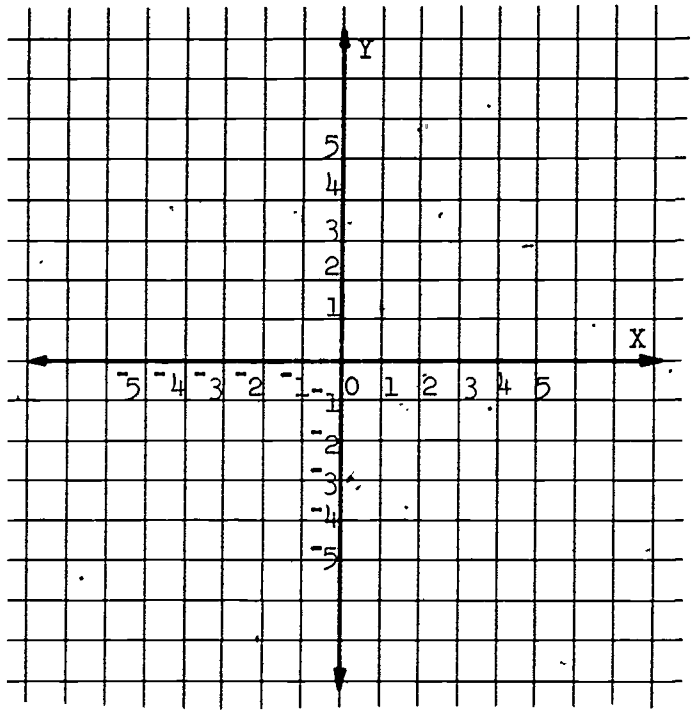
9. (Section 18-5)

Find the solution of these two equations by graphing them and writing the coordinates of their point of intersection.

$y = 3x + 2$

and

$y = 2x + 3$



The solution is

(___ , ___) .

10. (Section 18-6)

(a) What is the slope of the line $y = 4x + \frac{^-3}{4}$? ____________

(b) What is the reciprocal of the slope of the line $y = 4x + \frac{^-3}{4}$? ____________

(c) What is the opposite of the reciprocal for part (b)? ________

(d) Write the equation of the line that is perpendicular to $y = 4x + \frac{^-3}{4}$ and has a y-intercept of 6 . ____________

11. (Section 18-7)

Write the equation of the line that has a y-intercept of 9 and is parallel to the line whose equation is $y = \frac{3}{4}x + 5$.

$y =$ ____________

12. (Section 18-8)

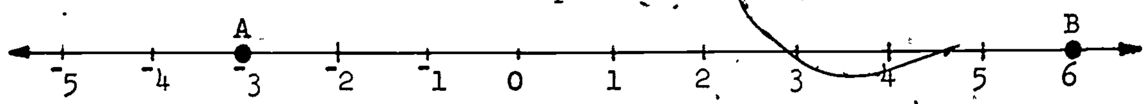
Find the value for each of the following:

(a) $|{}^{-}8| =$ ______ (c) $|3 - 7| =$ ______

(b) $|10| =$ ______ (d) $|{}^{-}5 - 10| =$ ______

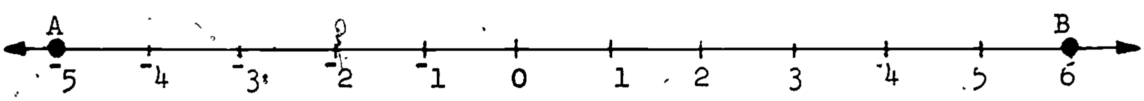
13. (Section 18-8)

What is the distance between points A and B ?



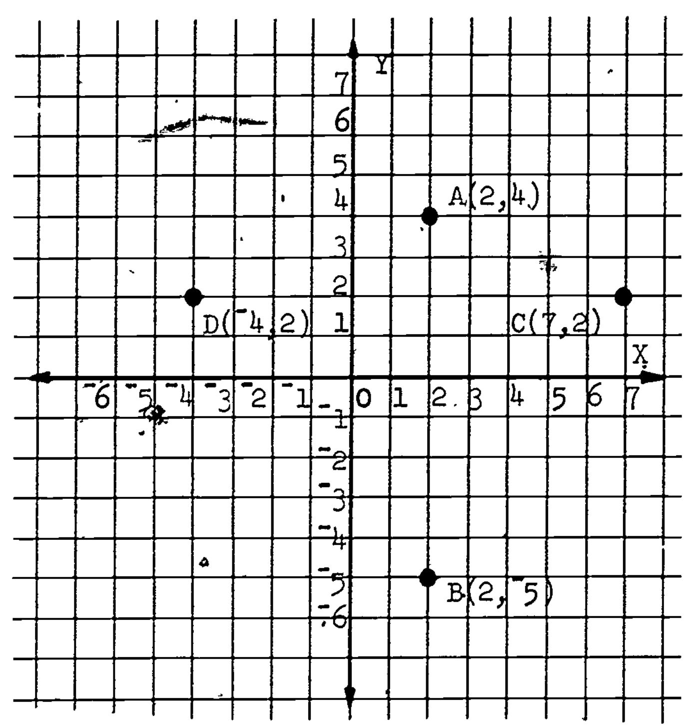
14. (Section 18-9)

What is the coordinate of the midpoint of $\overline{AB}$? ______



15. (Section 18-10)

Find the length of the following line segments.

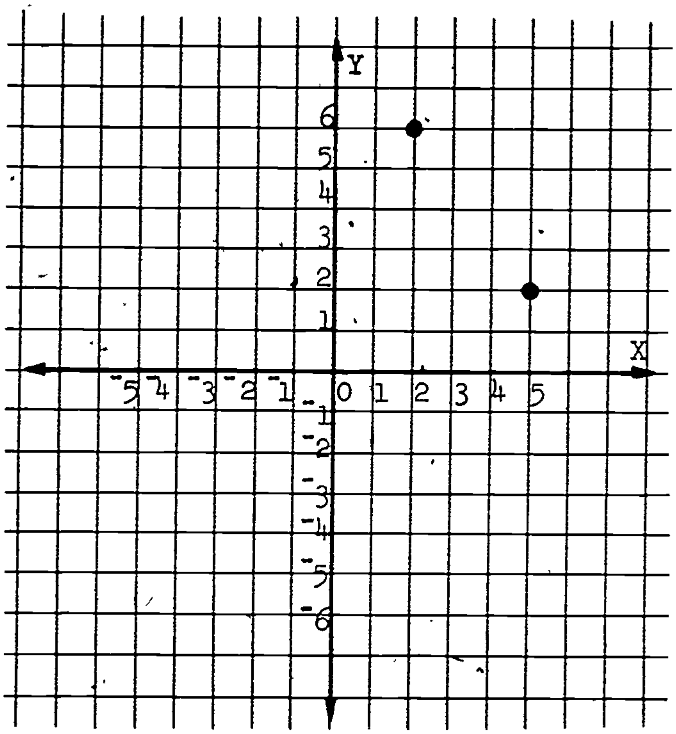


(a) $\overline{AB}$ ______

(b) $\overline{CD}$ ______

16. (Section 18-11)

Find the distance between point A and point B .



The length of $\overline{AB}$ is ______ .

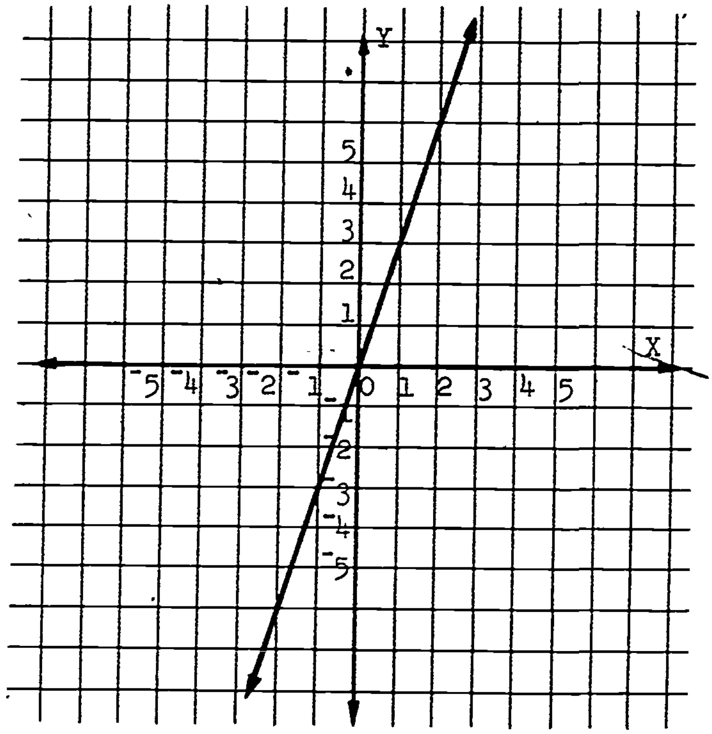
Test

1. For this graph:

   (a) What is the rise of the line? ________

   (b) What is the run of the line? ________

   (c) What is the slope of the line? ________



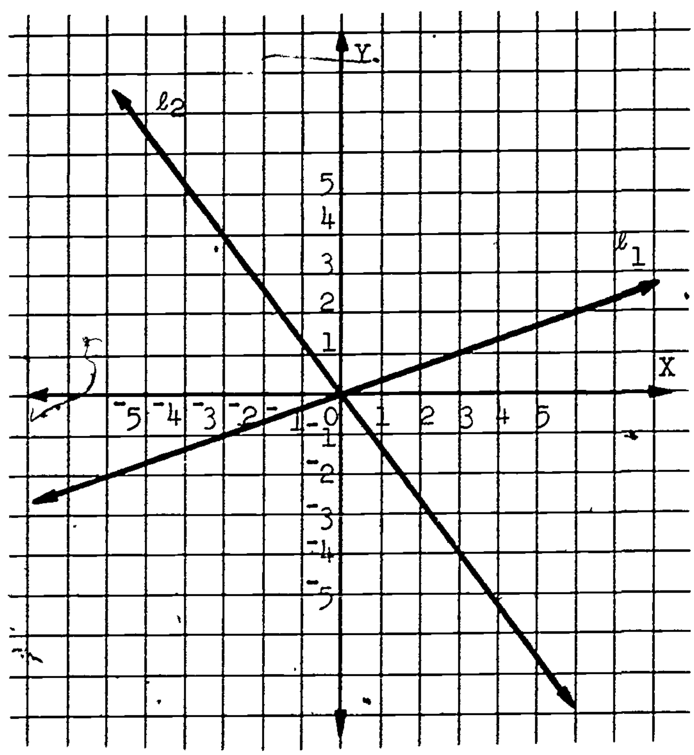
2. (a) Lines that lean to your right have a ____________ slope.

   (b) Lines that lean to your left have a ____________ slope.

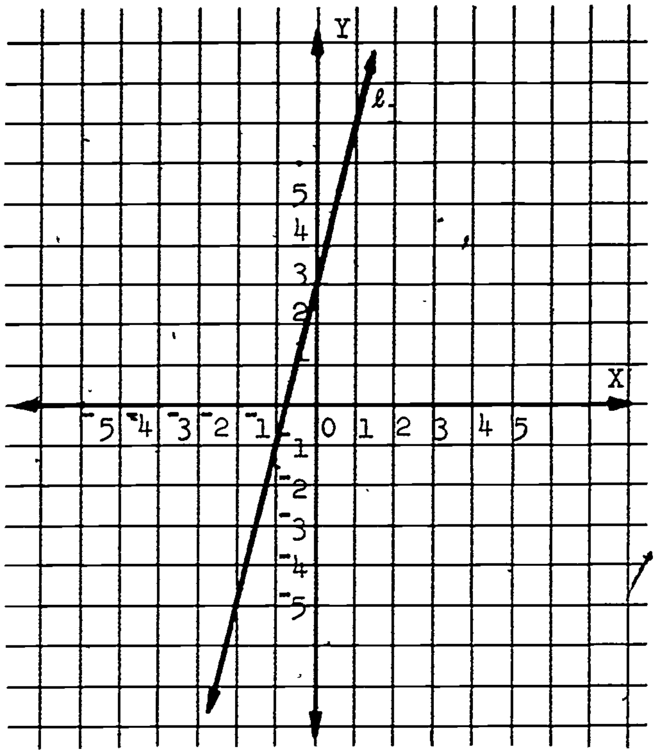
3. Here are two lines, ℓ_1 and ℓ_2.

   (a) What is the slope of ℓ_1 ? ________

   (b) What is the slope of ℓ_2 ? ________



4. Here is the graph of ℓ_1.

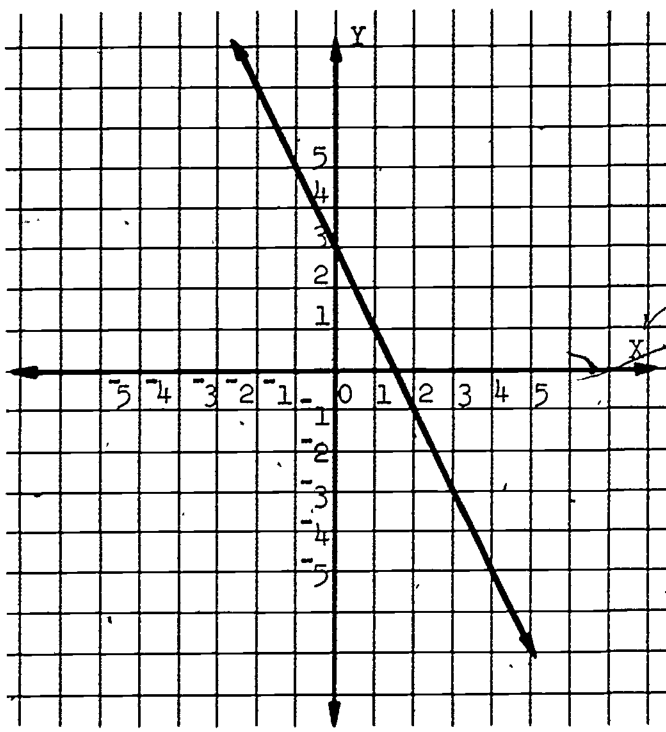


(a) What is the y-intercept of ℓ_1?

______

(b) What is the slope of ℓ_1? ______

5. Write the slope, the y-intercept, and the function for this line.



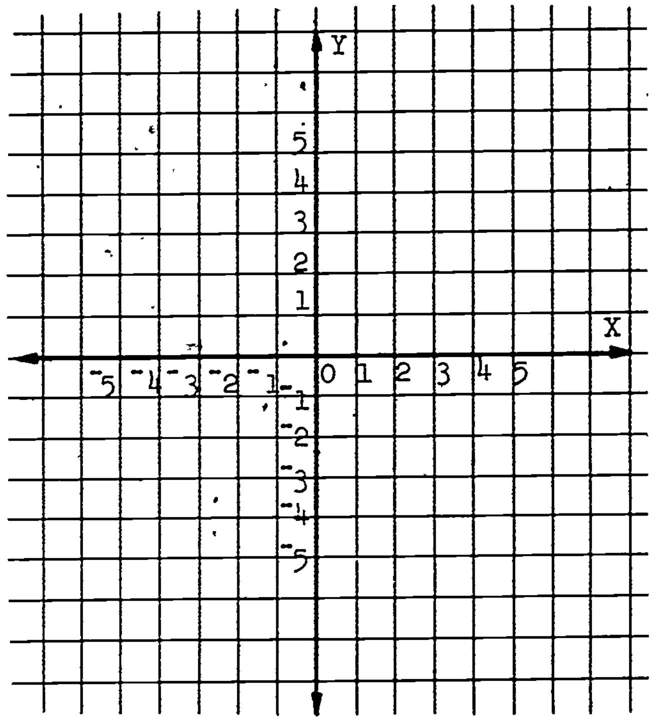
(a) Slope is ______.

(b) y-intercept is

______.

(c) $f : x \rightarrow$ ______

6. Without making a table of inputs and outputs, draw the graph of $f : x \rightarrow 2x + {}^{-}3$.



7. Here is the graph of ℓ_2 .

(a) What is the slope of this line? ______

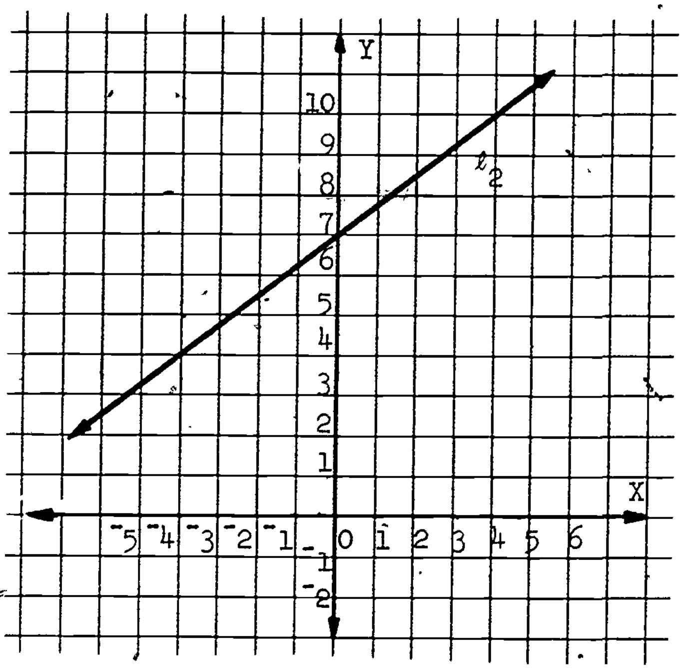
(b) What is the y-intercept of this line? ______

(c) Write the function for this line.

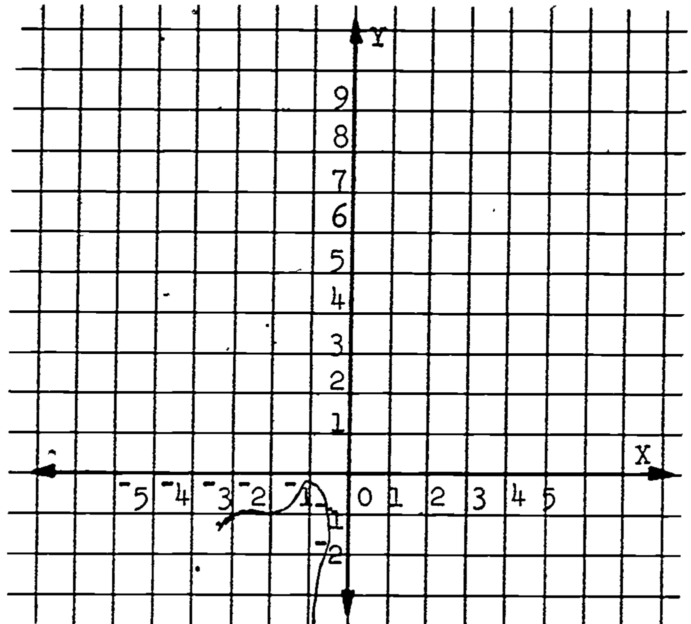
$f : x \rightarrow$ ____________

(d) Write the equation of this line.

$y =$ ____________



8. <u>Without</u> making a table of inputs and outputs, draw the graph of $y = \frac{4}{3}x + 5$.

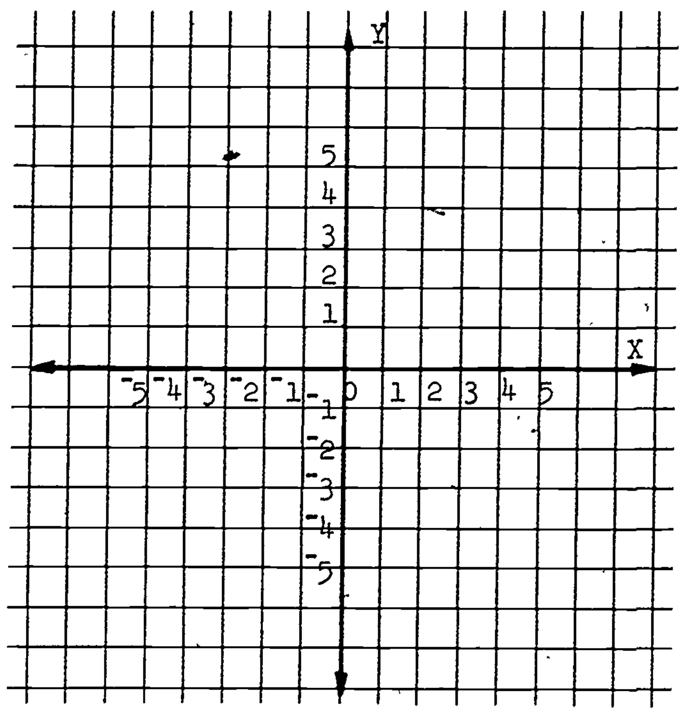


9. Find the solution of these two equations by graphing them and writing the coordinates of their point of intersection.

$y = {}^{-}2x + 1$

and

$y = 2x + {}^{-}3$



The solution is

(,).

10. (a) What is the slope of $y = 3x + {}^{-}5$. ______________

(b) What is the reciprocal of the slope? ________

(c) What is the opposite of the reciprocal for part (b)? ________

(d) Write the equation of a line that is perpendicular to

$y = 3x + {}^{-}5$ and has a y-intercept of 3 .

$y =$ ________________

11. Write the equation of the line that has a y-intercept of ${}^{-}5$

and is parallel to the line whose equation is $y = \frac{3}{2}x + 7$.

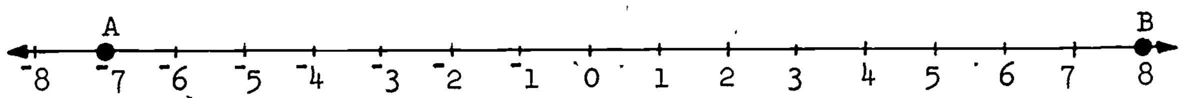
$y =$ ________

12. Find the value for each of the following:

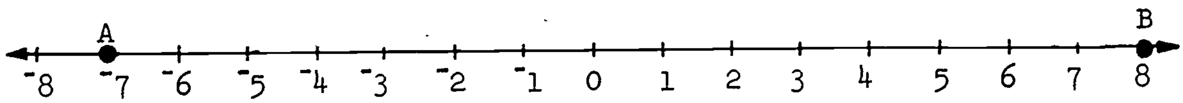
(a) $|{}^{-}12| =$ ________ (c) $|5 - 9| =$ ________

(b) $|8| =$ ________ (d) $|{}^{-}3 - 7| =$ ________

13. What is the distance between points A and B ? ________



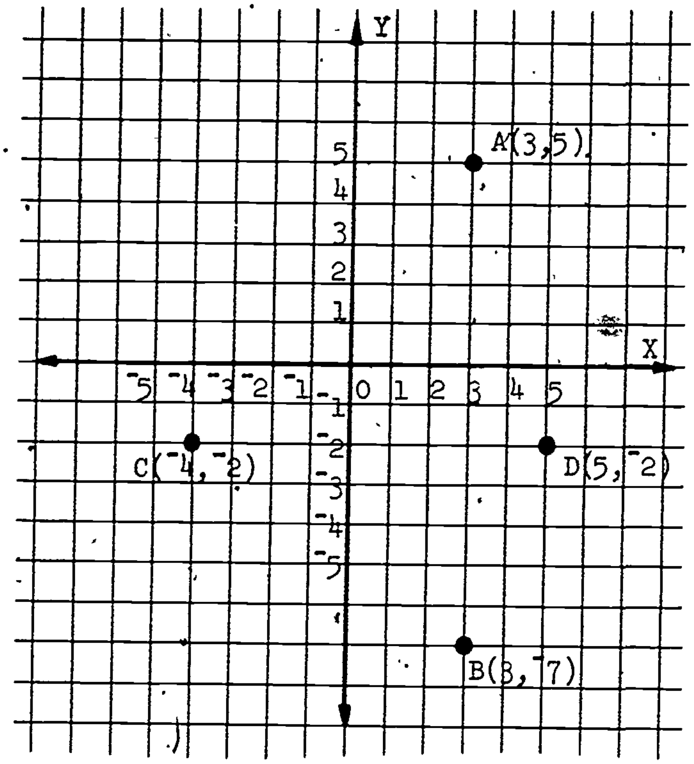
14. What is the coordinate of the midpoint of $\overline{AB}$? ________



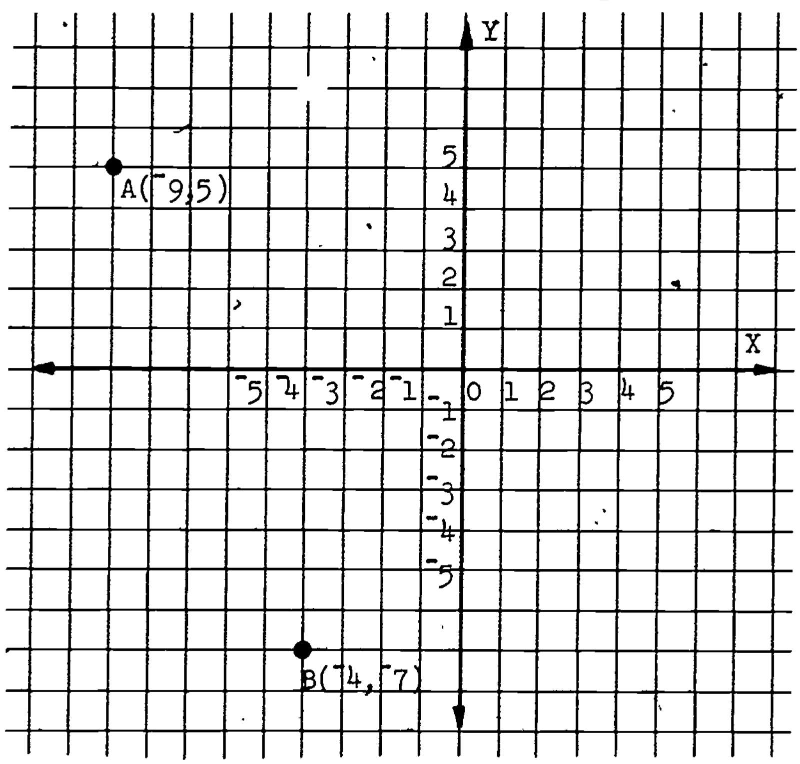
15. Find the length of the following line segments.

(a) Length of $\overline{AB}$ is ________.

(b) Length of $\overline{CD}$ is ________.



16. Find the distance between point A and point B.



The length of $\overline{AB}$ is ________.

Check Your Memory: Self-Test

1. (Section 13-6)

Fill the blanks. You may use your flow chart, page 7, if you need it. (Remember to rewrite subtraction problems as addition.)

(a) $\frac{3}{4} + \frac{7}{8} =$ ________

(b) $\frac{1}{3} + \frac{3}{4} =$ ________

(c) $\frac{7}{5} - \frac{1}{10} =$ ________

(d) $\frac{7}{8} - \frac{1}{2} =$ ________

2. (Section 13-11)

Fill the blanks.

(a) $\frac{1}{4} =$ ________ %

(b) .6 = ________ (Write a fraction.)

(c) .7 = ________ %

(d) 5% = ________ (Write a decimal.)

(e) $\frac{7}{8} =$ ________ (Write a decimal.)

(f) 130% = ________ (Write a decimal.)

(g) 75% = ________ (Write a fraction.)

(h) $\frac{3}{2} =$ ________ %

3. (Section 13-10)

Use scientific notation to write these.

(a) 34.063 = ____________

(b) .00982 = ____________

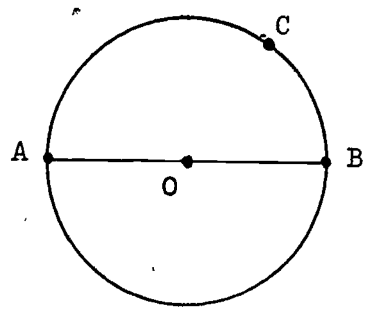
(c) 768,000 = ____________

(d) .015793 = ____________

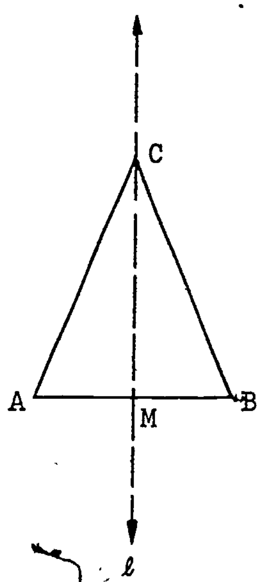
(e) 59764.31 = ____________

4. (Sections 14-4 and 14-5)

(a) For the circle below, O is the center and $\overline{AB}$ is the diameter. Construct a right angle with the vertex at point C .



(b) In the figure below, line ℓ is the perpendicular bisector of $\overline{AB}$.



$\overline{MA} \cong \overline{MB}$ because ______________________

$\angle AMC \cong \angle BMC$ because ______________________

$\overline{CM} \cong$ ________ because ______________________

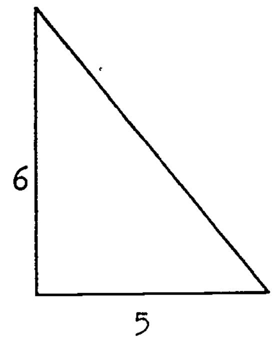
Therefore $\triangle AMC \cong \triangle$ _____ by the ________ congruence property.

What kind of triangle is $\triangle ABC$? ______________

5. (Section 16-4)

Find the length of the hypotenuse of each of these right triangles.

(a)



$$a^2 + b^2 = c^2$$

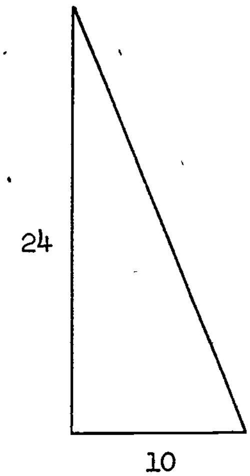
$$\underline{\quad\quad}^2 + \underline{\quad\quad}^2 = c^2$$

$$\underline{\quad\quad} + \underline{\quad\quad} = c^2$$

$$\underline{\quad\quad} = c^2$$

$$\underline{\quad\quad} = c$$

(b)



$$a^2 + b^2 = c^2$$

$$\underline{\quad\quad}^2 + \underline{\quad\quad}^2 = c^2$$

$$\underline{\quad\quad} + \underline{\quad\quad} = c^2$$

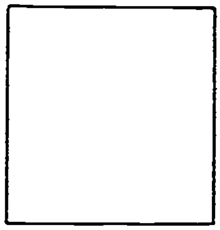
$$\underline{\quad\quad} = c^2$$

$$\underline{\quad\quad} = c$$

6. (Section 16-2)

Find the area of each square.

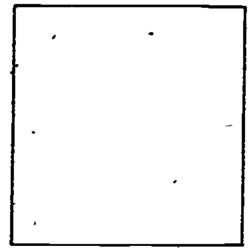
(a)



5

Area = ________

(b)

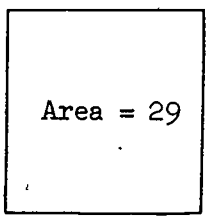


$\sqrt{27}$

Area = ________

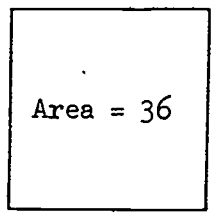
Find the length of the side of each square.

(c)



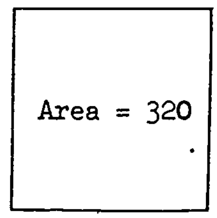
S = ______

(d)



S = ______

(e)



S = ______

7. (Section 16-10)

(a) Find the area of a circle whose radius is 10 inches.

(use $\pi \approx 3.14$) $A \approx$ ________ sq. in.

(b) Find the area of a circle whose radius is $\sqrt{14}$ ft.

(use $\pi \approx \frac{22}{7}$) $A \approx$ ________ sq. ft.

Now check your answers on the next page. If you do not have them all right, go back and read the section again.

Answers to Check Your Memory: Self-Test

1. (a) $\frac{13}{8}$

   (b) $\frac{13}{12}$

   (c) $\frac{13}{10}$

   (d) $\frac{3}{8}$

2. (a) 25%

   (b) $\frac{3}{5}$

   (c) 70%

   (d) .05

   (e) .875

   (f) 1.3

   (g) $\frac{3}{4}$

   (h) 150%

3. (a) 3.4063×10^{1}

   (b) 9.82×10^{-3}

   (c) 7.68×10^{5}

   (d) 1.5793×10^{-2}

   (e) 5.976431×10^{4}

4. (a) The easiest way is to draw $\overrightarrow{CA}$ and $\overrightarrow{CB}$.
$\angle ACB$ is a right angle.

(b) $\overline{MA} \cong \overline{MB}$ because ℓ bisects $\overline{AB}$.

$\angle AMC \cong \angle BMC$ because line $\ell \perp \overline{AB}$.

$\overline{CM} \cong \overline{CM}$ because it is the same segment.

$\triangle AMC \cong \triangle BMC$ by the SAS congruence property.

$\triangle ABC$ is an isosceles triangle.

5. $5^2 + 6^2 = c^2$
$25 + 36 = c^2$
$61 = c^2$
$\sqrt{61} = c$

$10^2 + 24^2 = c^2$
$100 + 576 = c^2$
$676 = c^2$
$26 = c$

6. (a) $A = 25$
(b) $A = 27$
(c) $s = \sqrt{29}$
(d) $s = 6$
(e) $s = \sqrt{320}$ (or $8 \cdot \sqrt{5}$ because $\sqrt{320} = \sqrt{64 \cdot 5}$
$= \sqrt{64} \cdot \sqrt{5}$
$= 8 \cdot \sqrt{5}$).

7. (a) $A \approx 314$ sq. in.
(b) $A \approx 44$ sq. ft.